Homework 1 - Solutions

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1. *Solution.* $\neg Q$ is true, therefore, Q is false. If Q is false, then $P \Rightarrow Q$ is true only if P is false. Therefore, $P \Rightarrow Q$ is true and $\neg Q$ is true implies that P is false. □

2. (a) *Solution.* Beer is an alcoholic drink. Joe is drinking beer. Therefore, we need to check if Joe is at least 21 years of age.

   Sandra is clearly over 21, therefore, she is allowed to drink alcoholic drinks. Hence, we don't need to check her drink.

   Milk is not an alcoholic drink. Peter is drinking milk. Therefore, it doesn't matter what Peter's age is.

   Cathy is obviously under 21. Therefore, Cathy cannot drink alcoholic drinks. Hence, we need to check Cathy's drink.

   As a result, we only need to check Joe's age and Cathy's drink. □

   (b) *Solution.* A is a vowel. Therefore, we need to turn over the card to check if an even number on the other side.

   2 is even. Therefore, it doesn't matter what is on the other side of the card.

   X is a consonant, i.e., not a vowel. Therefore, it doesn't matter what is on the other side of the card.

   3 is odd. Therefore, we need to turn over the card to check if we have a consonant or a vowel on the other side. □

3. (c) For all rational numbers x there exists a real number z such that $xz \neq 0$.

4. *Proof.* Assume d divides x. Then, there exists and integer y such that $x = dy$. Hence, $-x = -dy = d \cdot (-y)$. Since $-y$ is integer and $-x = d \cdot (-y)$, we have d divides $-x$. □

5. *Solution.* **Proof by means of a truth table**:

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

The column of truth values headed by $\neg(P \vee Q)$ is the same as the column of truth values headed by $\neg P \wedge \neg Q$. Therefore, $\neg(P \vee Q)$ is logically equivalent to $\neg P \wedge \neg Q$.

Proof by means of an explanation in words

$(\Rightarrow)$: Suppose $\neg(P \vee Q)$ is true. Then, the statement $P \vee Q$ is false, so both statements P and Q are false, so both statements $\neg P$ and $\neg Q$ are true, so the statement $\neg P \wedge \neg Q$ is true.

$(\Leftarrow)$: Suppose $\neg P \wedge \neg Q$ is true. Then, both statements $\neg P$ and $\neg Q$ are true, so both statements P and Q are false, so the statement $P \vee Q$ is false, so the statement $\neg(P \vee Q)$ is true.

Therefore, $\neg(P \vee Q)$ is logically equivalent to $\neg P \wedge \neg Q$. $\square$

6. *Proof.* For any real number a, $a \leq 0$ or $a \geq 0$. If $a \geq 0$, then $|a|^2 = a^2$. If $a \leq 0$, then $|a|^2 = (-a)^2 = a^2$. Hence, $|a|^2 = a^2$. $\square$