Homework 1

MAT 200, Instructor: Alena Erchenko

1. Suppose that $P \Rightarrow Q$ is true and $\neg Q$ is true. Explain why it follows that P must be false.

2. (a) Joe, Sandra, Peter, and Cathy are sitting at a table in a restaurant. Each of them has a glass of some beverage in front of them. Your job is to check whether or not they are legally entitled to be consuming the beverages that you see on the table in front of them. The drinking age is 21. Joe is drinking beer. Sandra is clearly over 21. Peter is drinking milk. Cathy is obviously under 21. Whose ages or beverages would you have to check? Explain your answer.

   (b) Four cards are lying on a table. Each card has a single letter on one side and a single number on the other side. The sides that are up show the following letters and numbers:

   $$A \qquad 2 \qquad X \qquad 3.$$

   Your job is to check whether or not the following rule holds: Whenever there is a vowel on one side of a card, then there must be an even number is on the other side. Which cards would you have to turn over to be sure that the rule holds? Explain your answer.

3. Consider the following statement:

   There exists a rational number x such that for all real numbers z, $xz = 0$.

   Which of the following statements is the negation of the above statement?

   (a) There exists a non-rational number x such that for all real numbers z, $xz = 0$.
   (b) There exists a non-rational number such that there exists a real number z such that $xz \neq 0$.
   (c) For all rational numbers x there exists a real number z such that $xz \neq 0$.
   (d) For all rational numbers x there exists a non-real number z such that $xz \neq 0$.
   (e) For all rational numbers x there exists a non-real number z such that $xz = 0$.

4. Let $d, x \in \mathbb{Z}$. Prove that if d divides x, then d divides $-x$.

5. Prove the following theorem in two ways: by means of a truth table and by means of an explanation in words.

   Theorem 0.1. *Let P and Q be statements. Then,*

   $$\neg(P \vee Q) \textit{ is logically equivalent to } \neg P \wedge \neg Q.$$

6. Prove that $|a|^2 = a^2$ for every real number a, where $|a|$ denotes the absolute value of a, i.e. $|a| = a$ if $a \geq 0$ and $|a| = -a$ if $a \leq 0$.