

Homework 2

MAT 200, Instructor: Alena Erchenko

1. Prove that for all real numbers a and b ,

$$|a| \leq |b| \Rightarrow a^2 \leq b^2.$$

2. Negate the following statements.

(a) There exists a rational number q such that for all positive real numbers x , $\frac{q}{x} < 0$.

(b) $x > 0$ or $\frac{x}{2}$ is a natural number.

(c) For all integer numbers y there exists a rational number s such that $y - s > 0$ and $ys = 43$.

3. Prove that for all real numbers a , b , and c , we have

$$bc + ac + ab \leq a^2 + b^2 + c^2.$$

4. Let x and y be integers. Prove that if x is even and y is odd then $x + y$ is odd.

5. Show that there are no natural numbers x and y such that $x^2 - y^2 = 1$.

6. Using a proof by cases, show that, for all real numbers a and b ,

$$|a + b| \leq |a| + |b|.$$