Homework 4

MAT 200, Instructor: Alena Erchenko

1. Let u_n be the nth Fibonacci number. Prove, by strong induction on n (without using the formula for u_n that we proved in class), that

$$u_{m+n} = u_{m-1}u_n + u_m u_{n+1}$$

for all natural numbers $m \geq 2$ and $n \geq 1$.

2. Prove that if $X = \{1, 2, 3\}$ and $Y = \{\text{solutions of } x^3 - 2x^2 - x + 2 = 0\}$, then $X \not\subset Y$.

3. Prove or disprove by providing counterexamples the following statements:

   (a) If $(A \setminus B) \neq \emptyset$, then $A \neq B$.

   (b) If $A \neq B$, then $A \setminus B \neq \emptyset$.

   (c) If $C \not\subset A$, then $C \not\subset (A \cup B)$.

4. Show that for all sets A, B, C, we have $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

5. Show that a set of n elements has 2^n subsets for $n \in \mathbb{N}$.

6. Prove that for each $n \in \mathbb{N}$, there exists a prime number q such that $n < q \leq 1 + n!$.

   Hint: Use proof by contradiction, lemma that for natural numbers a, b if a divides b, then $a \leq b$, definition of a factorial.