Homework 5

MAT 200, Instructor: Alena Erchenko

1. Prove that for every integer x, if x is odd then x^3 is odd.
2. Prove that for every integer x, if x is odd then there exists an integer y such that $x^2 = 8y + 1$.
3. Proof that for all sets A, B, and C, we have $(A \cap B) \times C = (A \times C) \cap (B \times C)$.
4. (DeMorgan's Laws) Prove that if $\{A_n\}_{n\in\mathbb{N}}$ is a collection of subsets of A, then

   $$\left(\bigcup_{n\in\mathbb{N}} A_n\right)^c = \bigcap_{n\in\mathbb{N}} A_n^c$$

   and

   $$\left(\bigcap_{n\in\mathbb{N}} A_n\right)^c = \bigcup_{n\in\mathbb{N}} A_n^c$$

   where the complements are taken relative to A.
5. For any $n \in \mathbb{N}$ define $A_n = (-\frac{1}{n}, \frac{1}{n}) = \{x \in \mathbb{R} | -\frac{1}{n} < x < \frac{1}{n}\}$. Find $\bigcap_{n\in\mathbb{N}} A_n$ and justify your answer with a proof.
6. Let $E \subset \mathbb{R}$ and $F \subset \mathbb{R}$. Let $X = \{(x, y) | x, y \in \mathbb{R} \text{ and } y = x\}$. Prove that if $X \subset E \times F$, then $E \times F = \mathbb{R}^2$.