Homework 7

MAT 200, Instructor: Alena Erchenko

1. Let x and y be real numbers. Prove that if x is rational and y is irrational, then $x + y$ is irrational.

2. Prove that $\sqrt{3}$ is an irrational number.

3. Let x be a real number such that

$$x^3 + ax^2 + bx + c = 0,$$

where $a, b, c \in \mathbb{Z}$.

   (a) Prove that if x is rational, then x is an integer.

   (b) Prove that if x is not an integer, then x is irrational.

4. Let $a, c \in \mathbb{Z}$ and $b \in \mathbb{N}$ such that $GCF(a, b) = 1$. Prove that for all $n \in \mathbb{N}$, we have that if a divides cb^n, then a divides c.

5. Let n be an integer. Prove that 3 divides n and 5 divides n if and only if 15 divides n.

6. Find the prime factorizations of 136, 150, 255, and 1980. Then, compute $GCF(136, 150)$ and $GCF(255, 1980)$, where GCF stands for the greatest common factor. (No proof is required here, just write down the answers.)