Homework 8

MAT 200, Instructor: Alena Erchenko

1. For each of the following relations, denoted by R, on the set X determine whether it is reflexive, whether it is symmetric, whether it is transitive, and whether it is an equivalence relation.

   (a) For $X = \mathbb{Z}$, put aRb if and only if $ab \geq 0$;

   (b) For $X = \mathcal{P}(Y)$ for some set $Y \neq \emptyset$, put ARB if and only if $A \subset B$.

   (c) For $X = \mathbb{N}$, put nRm if and only if $n + m$ is a prime.

2. Let $n \in \mathbb{Z} \setminus \{0\}$. Show that the congruence modulo n is an equivalence relation on $\mathbb{Z}$.

3. Let $a, b, c \in \mathbb{Z}$ and $n \in \mathbb{Z} \setminus \{0\}$. Prove that if $a \equiv b \pmod{n}$, then $ca \equiv cb \pmod{n}$.

4. Let $a_1, b_1, a_2, b_2 \in \mathbb{Z}$ and $n \in \mathbb{Z} \setminus \{0\}$. Suppose $a_1 \equiv b_1 \pmod{n}$ and $a_2 \equiv b_2 \pmod{n}$. Show that $a_1 + a_2 \equiv b_1 + b_2 \pmod{n}$.

5. Are the following statements true or false? Explain your answer.

   (a) $9^{73} \equiv 1 \mod 8$.

   (b) $14^{198} - 2 \equiv 5 \mod 7$.

6. Suppose that a positive integer n is written in decimal notation as $n = a_3a_2a_1a_0$ where for all $i \in \{0, 1, 2, 3\}$ we have $0 \leq a_i \leq 9$. Prove that n is divisible by 11 if and only if the alternating sum of its digits $a_0 - a_1 + a_2 - a_3$ is divisible by 11.