Homework 9

MAT 200, Instructor: Alena Erchenko

1. Does there exists $x \in \mathbb{Z}$ such that $15x \equiv 5 \pmod 3$? Explain your answer.
2. Consider a function $f\colon \mathbb{R} \to \mathbb{R}^2$ defined as $f(x) = (3x^2, x^2 - 3)$ for all $x \in \mathbb{R}$. Determine whether f is injective and whether f is surjective and explain your answer.
3. Consider a function $f\colon \mathbb{R} \to \mathbb{R}^2$ defined as $f(x) = (3x, x^2 - 3)$ for all $x \in \mathbb{R}$. Determine whether f is injective and whether f is surjective and explain your answer.
4. Let A and B be sets, let f be a function on A, and let g be a function on B. If we try to define a function on $A \cup B$ by letting
$$h(x) = \begin{cases} f(x) \text{ if } x \in A, \\ g(x) \text{ if } x \in B, \end{cases}$$
what do we need to know about f and g to be sure that h is a well-defined function?
5. Prove $f(x, y) = (x - y, 2x)$ defines a bijective function $\mathbb{R}^2 \to \mathbb{R}^2$.
6. Give an example of sets X and Y and functions $f\colon X \to Y$, $g\colon Y \to X$ such that $g \circ f = id_X$, yet f isn't surjective and g isn't injective.