Physics of Networks

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WHAT WE ARE GOING TO TALK ABOUT?

- What is called a Complex system ?
- What is Social physics ?
- An introduction to networks and graph theory
- degree distribution in Networks
- Resilience to the removal of nodes
- Spreading processes on Networks
- The small-world effect
- Correlation and communities

COMPLEX SYSTEM

- The most common and significant properties of complex systems are :

  (i) The system is composed of a large number of interacting elements;

  (ii) The interaction among the elements are nonlinear;

  (iii) Each element is unaware of the behavior of the system as a whole , it reacts only to locally available informations ;

  (iv) The system is usually open and in a state far from equilibrium ; and

  (v) Complex system have a history , their actual and future behavior depend upon this history and are particularly sensitive to it .

COMPLEX SYSTEM

- The toolbox available to study Complex systems drives from three main areas :

  1. Nonlinear dynamics
  2. Statistical Physics
  3. Network Science

- Some examples of Complex systems are :

  1. Biological systems
  2. Colonies and Communities of living things (including human societies)
  3. Financial systems (Stock markets , ...)

and many other systems .

Image Source: Wikipedia

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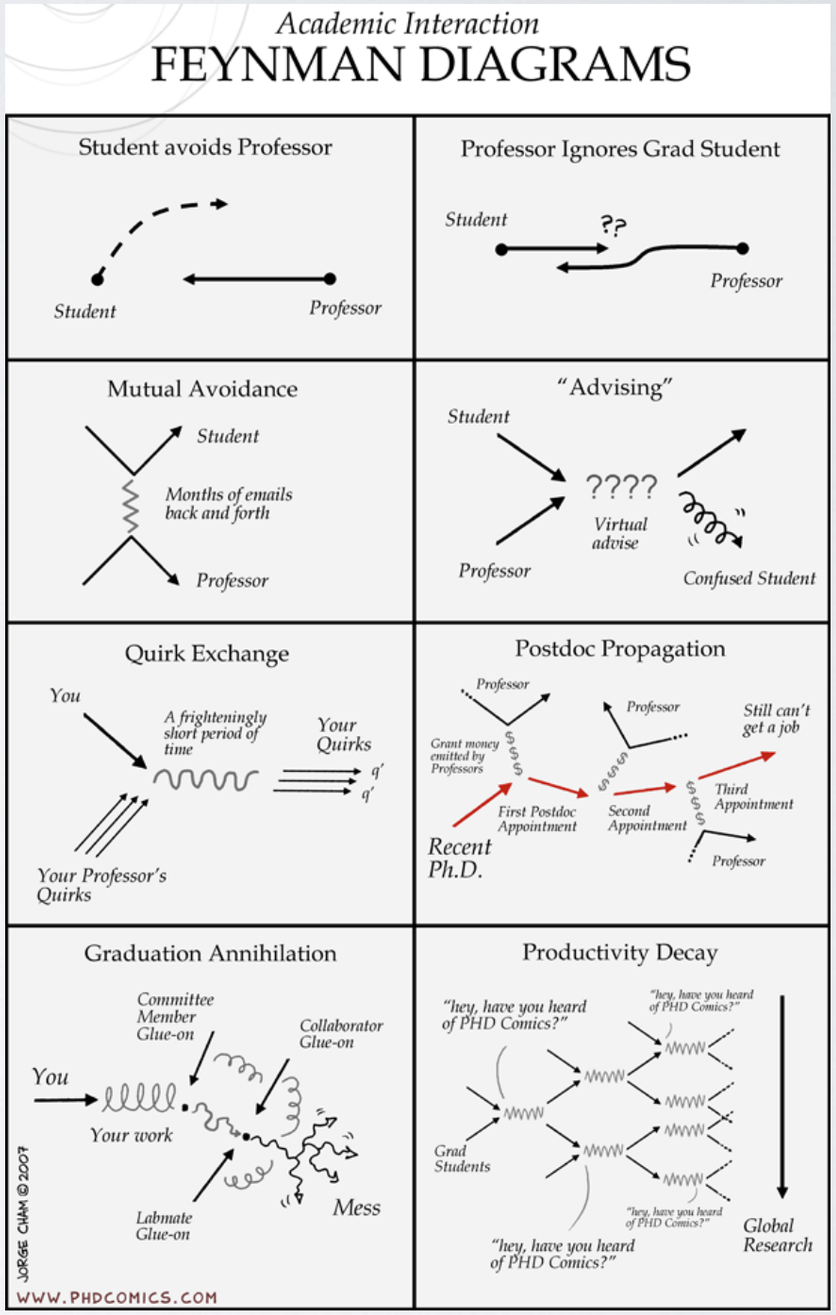
SOCIAL PHYSICS

- **social physics** is the study of social systems and social phenomena from the perspective of physics . Said another way, social physics is "the application of the concepts of physics in the social science" .

- Mathematical tool we use to model a society is a Network .



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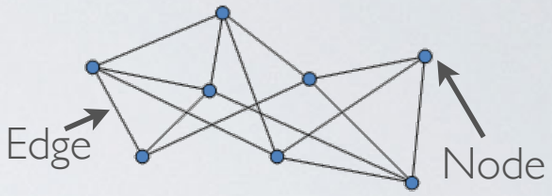
NETWORKS

- In its simplest form , a network is a collection of points, or 'nodes' , joined by lines, or 'edges' .



- As purely mathematical objects, networks (or so-called graphs) have been the subject of academic scrutiny since at least the 18th century . But they have taken a new practical role in recent years as a primary tool in the study of complex systems .

- There are lots of complex systems that can be represented as a network ; Like : Internet, World Wide Web , Social networks of friendships between individuals , and many other systems.

- Networks are also increasingly common in the study of Biological networks , epidemiology , computer viruses , genetics , human transportation and communication , and many other things .

- "Leonhard Euler" is often credited with the first rigorous result in graph theory , with his solution of the famous 'Königsberg bridge' problem in 1765 . Kenneth Appel and Wolfgang Haken's 1976 proof of the 'four-color' theorem is perhaps the best known recent achievement of graph theory .

Figure 1. A network representation of the internet. Each node represents an autonomous system—a group of computers under single administrative control, such as the computers at a university or a corporation. Edges (connecting lines) represent direct peering relations between autonomous systems, which are a rough indicator of where optical fibers and other data connections run. (Patents pending; © Lumeta Corp 2007. All rights reserved.)

Image Source: The Physics of Networks, Mark Newman, Physics today 35, 2008.

- From looking at the network, it is evident that although the pattern of connections is not a regular one, neither is it completely random.

DEGREE DISTRIBUTION IN A NETWORK

- The degree of a node in a network is the number of edges connected to that node .
- The degree distribution is one of the most basic quantitative properties of a network .
- Networks in which node's degree follow a poisson distribution ($p(k) \approx \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$) are called 'ER' (**E**rdös-**R**ényi) networks.
- Networks that follow a power law degree distribution ($P(k) \propto k^{-\alpha}$) are called 'Scale free' .

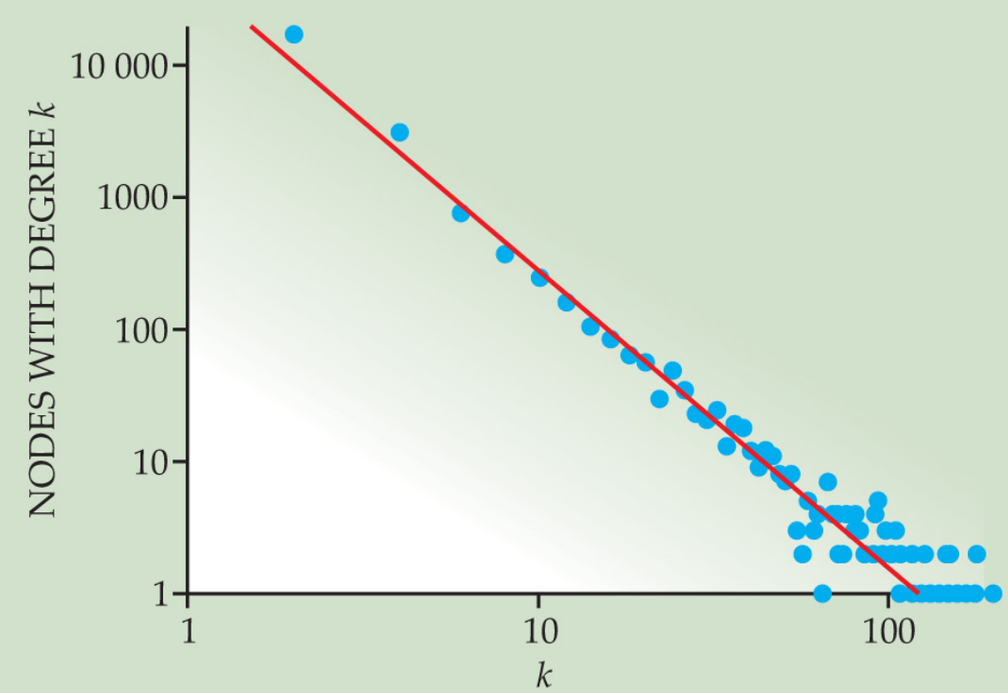


Figure 2. The distribution of the degrees of nodes on the internet. As indicated, the distribution roughly follows a straight line on a logarithmic plot; that is, it obeys a power law.

Image Source: The Physics of Networks, Mark Newman, Physics today 35, 2008.

RESILIENCE TO THE REMOVAL OF NODES

- Question 1 : by some estimates , as many as 3% of routers world wide may be nonfunctional at any given time ; why internet network always stays connected though ?

- Question 2 : networks like internet are fragile to removal of hubs (removing only 3% of nodes is sufficient to disconnect the network) . is this fragility always a negative point ?

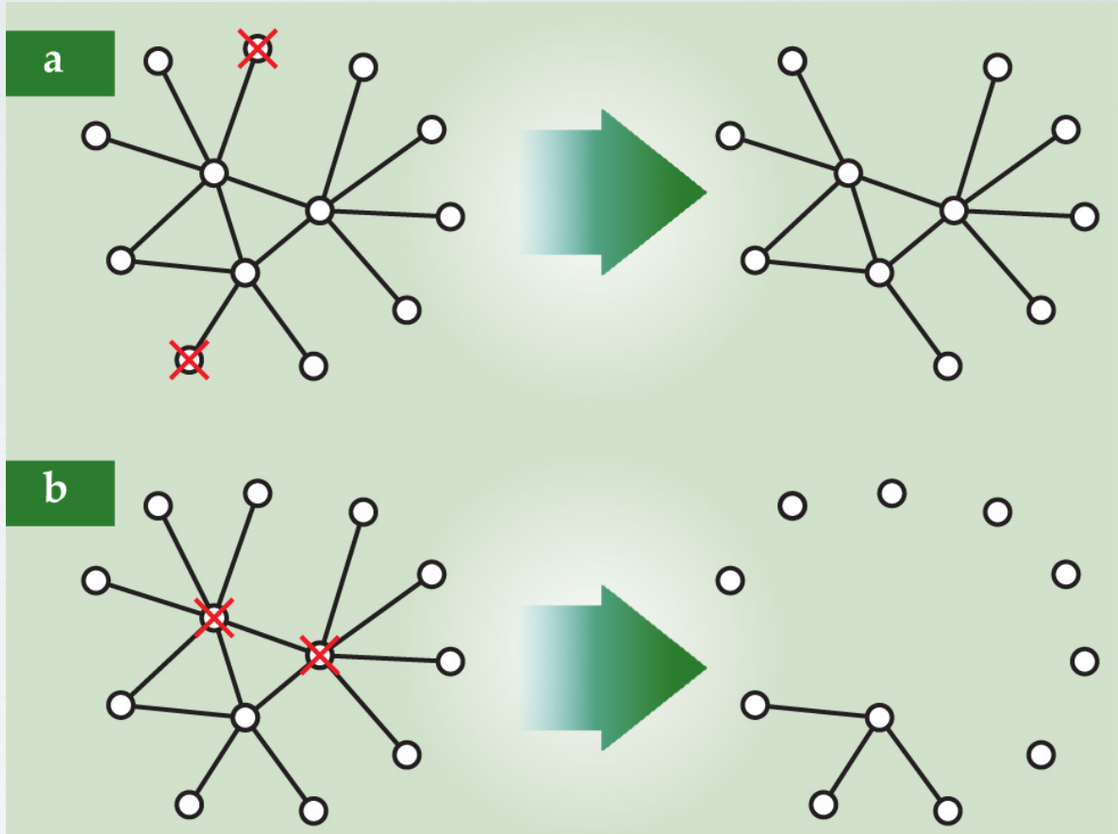


Figure 3. (a) Random removal of nodes from a scale-free network typically has little effect on the overall connectivity. **(b)** Targeting the highest-degree nodes can have a devastating effect.

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SPREADING PROCESSES ON NETWORKS

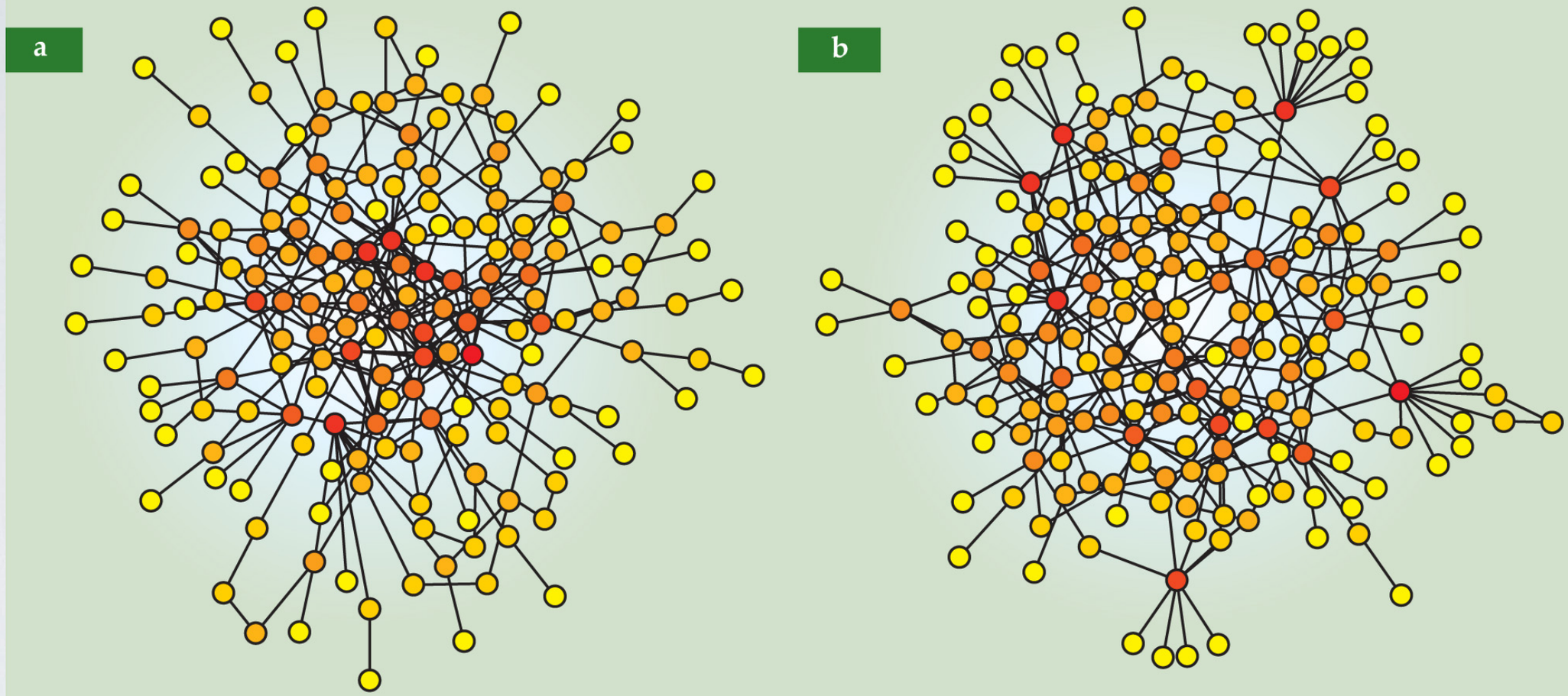
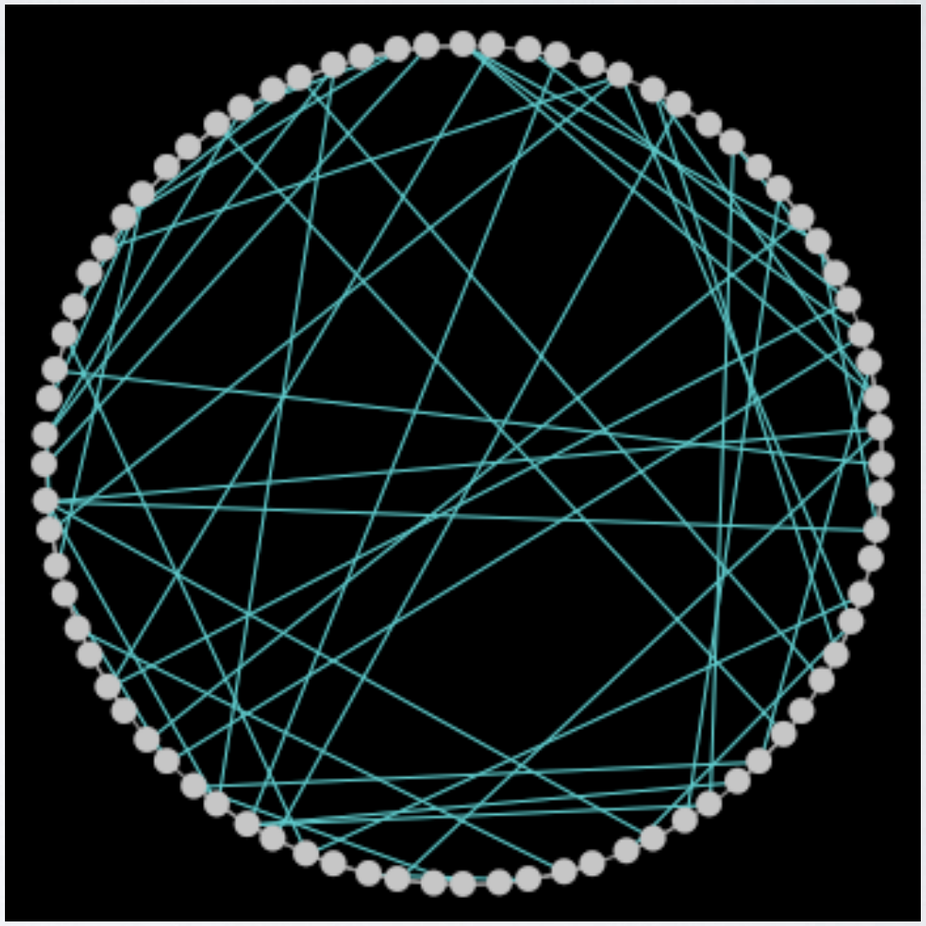


Figure 4. Two networks with the same distribution of node degrees but different degree correlations. Network **(a)** is positively correlated, whereas **(b)** is negatively correlated. The nodes are color coded to emphasize their degrees (yellow for low degree, red for high degree). The high-degree nodes clump together in network a but are more spread out in network b. A number of the hub-and-spoke formations characteristic of negatively correlated degrees—created when a high-degree node connects to a lot of low-degree ones—are visible around the edges of network b.

Image Source: The Physics of Networks, Mark Newman, Physics today 35, 2008.

SMALL-WORLD EFFECT

- small world effect : most pairs of people , no matter how distant they may be , are connected by at least one and probably many short chains of acquaintances .



network of 80 nodes with rewiring probability of 0.3 and average path length of 3.663

Image Source: Wikipedia

CORRELATION AND COMMUNITIES

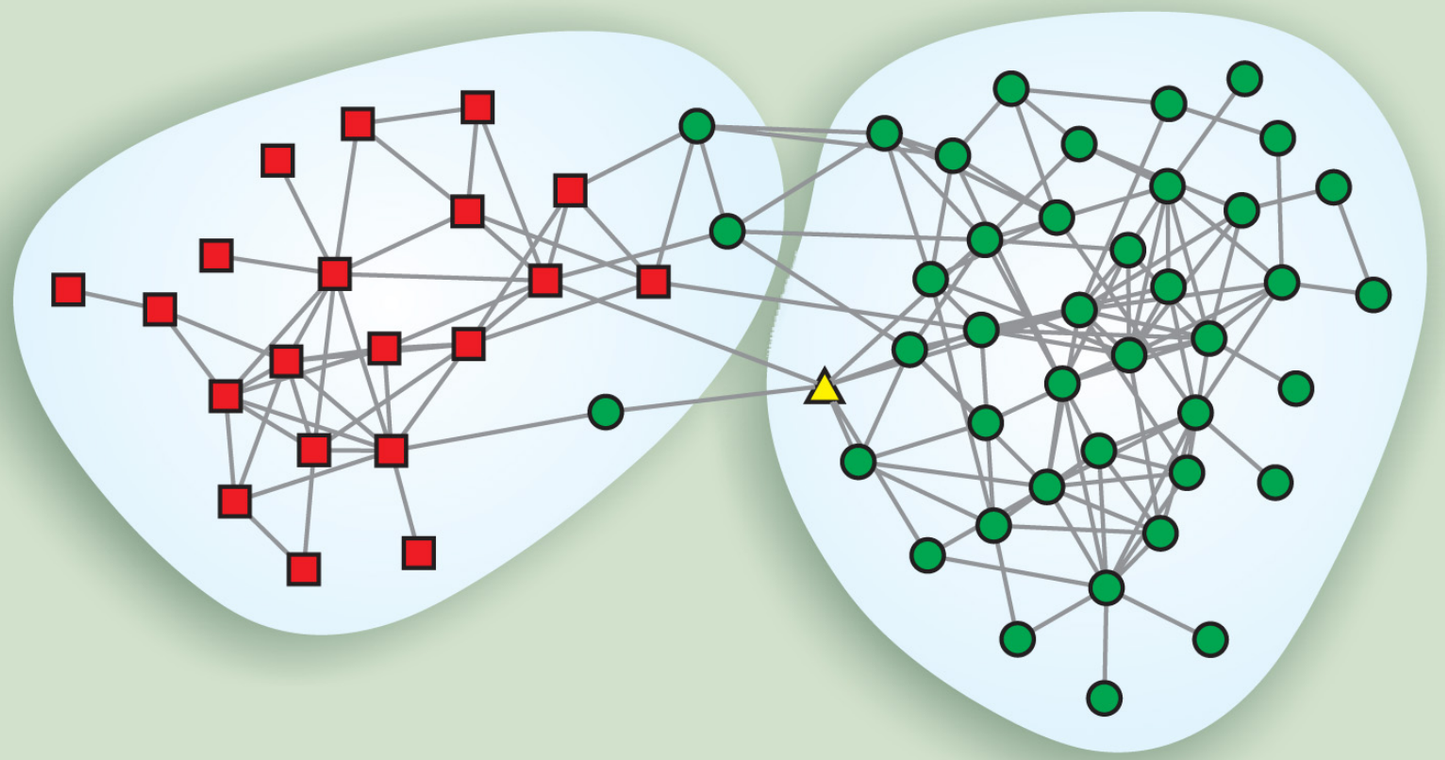
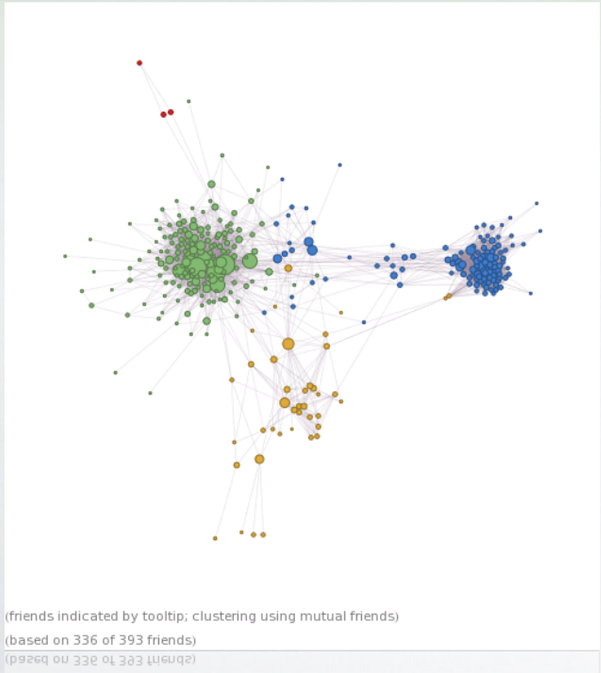
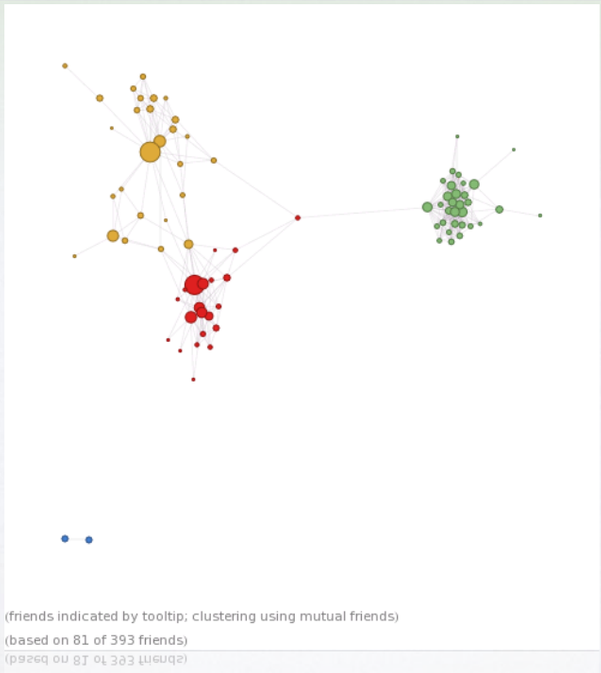
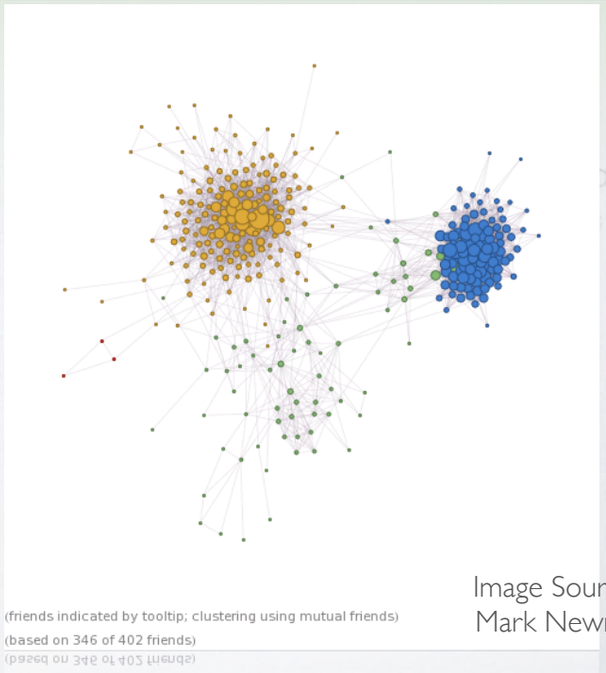


Figure 5. The social network of a community of bottlenose dolphins. The node shapes and colors indicate the two groups into which the network split upon the departure of the dolphin denoted by the triangle. The two circled regions indicate the division of the network found by an automated network analysis technique that considered only the pattern of connections.







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THANKS