P1.

EM comps.

2014F EM1.

(a). $t = \frac{x}{c}$ $\quad \omega t = \frac{\omega}{c} x \equiv \frac{x}{R_0}$ determines.

☆ (b). far field. use equations: $V \propto \frac{1}{r}$. $\vec{A} \propto \frac{1}{r}$.

P446. V and $\vec{A}$ form.

$\vec{E} = -\nabla V - \partial_t \vec{A} \propto 1/r$. $\quad i\vec{k} \times \vec{B} = \cancel{\mu_0} \partial_t \vec{E} \Rightarrow \vec{B} \sim \vec{E} \propto 1/r$.

(c). $i\vec{k} \times \vec{B} = \frac{1}{c}\cancel{\partial_t} \, i\omega \vec{E} \Rightarrow kB = \frac{\omega}{c} E \Rightarrow \cancel{E = B}$ $E/B = 1$.

(d). $\vec{S} = \frac{1}{\mu_0}(\cancel{\vec{B} \times \vec{E}}\ \vec{E} \times \vec{B}) = \frac{1}{\mu_0}(\mu_0 \vec{k} \times \vec{B}) \times \vec{B} = \vec{k} B^2 = \vec{k}(i\vec{k} \times \vec{A})^2 \sim k^3 \cdot A^2 = \ldots\ldots$

(e). "static" ~~fied~~ field.

$V \propto \frac{1}{r^2} \Rightarrow E \propto 1/r^3 \Rightarrow \frac{1}{r} B = \frac{\omega}{c} E \Rightarrow B \propto 1/r^2$

(f) $E/B = R_0/r$.

☆. (g) . . .

some $\partial_\theta(\vec{e}_\phi)$ formulas

2014F EM2

☆. (a). $\vec{B} = \frac{1}{4\pi r^3}[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}]$ outside

solution trick or P46 on P519!

$\vec{n} \times (\Delta \vec{B}) = \vec{K}_b$ and $\vec{n} \cdot \Delta\vec{B} = 0$. and $\vec{m} = \int \vec{M} d\tau$. and $\vec{K} = \vec{M} \times \vec{n}$.

$\Rightarrow \vec{B} = \frac{2}{3}\mu_0 \vec{M}$. Inside.

(b). $\vec{K} = \vec{M} \times \vec{n} = K_0 \hat{\phi}$, $K_0 = \phi \cdot (B_{out} - B_{in})$

(c) $d\vec{F} = I d\vec{l} \times \vec{B}$ where $d\vec{l} = \vec{e}_\theta \, dl$, $\vec{B} = B_1 \hat{r} + B_2 \hat{z} \Rightarrow -\vec{e}_\phi$

$\Rightarrow dF = I \cdot \cancel{r d\theta}\, dl \cdot B_r = I \cdot b \, d\theta \cdot \frac{2m \cos\theta}{4\pi b^3} = \frac{m}{2\pi b^2} I \cos\theta \, d\theta$

$\cancel{\tau = \int \vec{r} \times d\vec{F} = \int b \, dF = 0}$. $\vec{\tau} = \int \vec{r} \times d\vec{F}$. $\Rightarrow$ along x

(d). $\vec{\tau} = \int \vec{r} \times dF(-\vec{e}_\phi) = -\int r \, dF \, \hat{r} \times \vec{e}_\phi = \int \vec{e}_\theta \, r \, dF = \frac{mI}{2\pi b} \int \cos\theta \, \vec{e}_\theta \, d\theta$

$\int \cos^2\theta \, d\theta = \pi/2$ $\quad \int \cos\theta \sin\theta \, d\theta = 0. \Rightarrow \int \cos\theta(\cos\theta, \sin\theta, 0) \, d\theta$

$= (\pi/2, 0, 0)$.

$\Rightarrow \tau = \frac{mI}{2\pi b} \cdot \pi/2$

P2.

$$\left\{\begin{array}{l} x' = \cancel{vt}\ 0 \\ y' = b \\ z' = 0 \end{array}\right. \Rightarrow \left\{\begin{array}{l} t^* = \gamma t' - \gamma\beta x' = \gamma t \\ x^* = \gamma\beta t' + \gamma x' = \gamma\beta \\ y = y' = b \\ z = z' = 0 \end{array}\right.$$

☆ P531 ' refers to the moving, no ' refers to observer (static)

2014F EM7.

(a). $\vec{E}_0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}(\overset{-vt''}{\cancel{x}}, b, 0)$.

$\cancel{E_{x0} = 0 = E_{x0}}$. $\cancel{E_{y0} = \gamma}$. $E_{z0}' = 0 = \gamma \cdot E_{z0}$

$E_y = \gamma E_y' = \frac{1}{4\pi\varepsilon_0} \cdot q \cdot b \cdot \frac{1}{r^3}(\gamma) \cdot b \ldots$

$E_x = E_x' = \frac{1}{4\pi\varepsilon_0} \cdot q(-v) \cdot \gamma t \quad \ldots \quad B_z = \gamma \beta E_y$

$B_x = B_x' = 0. \quad B_y = 0 \quad \cancel{B_z = 0}$

☆ what's the direction?

(b). $E_y = C \cdot b^2 \cdot \frac{1}{(\gamma^2 t^2 + b^2)^{3/2}} \cdot \gamma \propto \sqrt{\frac{b^4\gamma^2}{(\gamma^2t^2+b^2)^3}} \overset{t>0}{=} -\frac{q\gamma}{b^2}$

$E_x = \frac{1}{4\pi\varepsilon_0} q \cdot (-v) \cdot \ldots$

(c) $E_y / E_x = \gamma \cdot E_y' / E_x' = \cancel{\gamma}\ \cancel{vt/b}\ b/vt \Rightarrow \cancel{b > v/t}\ b > vt$.

★. the solution impulse

(d). $\frac{F \cdot \Delta t}{m} \cdot \Delta t < b \qquad F = qE_{y\max} \Rightarrow m > \frac{F\Delta t^2}{b} = \frac{q}{b} \cdot \frac{\gamma q}{b} \cdot \underbrace{\boxed{\frac{b^2}{\gamma^2 v^2}}}_{\Delta t} \cdot \ldots$

(e). $u = \frac{F \cdot \Delta t}{m} = \ldots\ldots$

2015S EM1

(a). they must be "no source" part of Maxwell. where

$\nabla \times \vec{E} = -\frac{1}{c}\partial_t \vec{B}$ and $\nabla \cdot \vec{B} = 0$.

comes from $\vec{B} = \nabla \times \vec{A} \qquad \vec{E} = -\nabla\varphi - \frac{1}{c}\partial_t \vec{A}. \qquad \nabla \cdot \vec{A} + \frac{1}{c}\partial_t \varphi = 0.$

$\Rightarrow \left\{\begin{array}{ll} \nabla \cdot \vec{E} = \rho & \text{reads} \quad \Box\varphi = -\rho. \\ \nabla \times \vec{B} = \frac{1}{c}\vec{J} + \frac{1}{c}\partial_t \vec{E} & \text{reads} \quad \Box\vec{A} = -\vec{J}/c \end{array}\right.$

☆. (b). for $\Box G = \delta \Rightarrow \Box\varphi = \alpha \Rightarrow \varphi = \int G * \alpha$.

$\Rightarrow \varphi = \theta \int \frac{1}{4\pi|\vec{r} - \vec{r}_0|} \cdot \delta\left(t - t_0 - \frac{|\vec{r} - \vec{r}_0|}{c}\right) \cdot q \cdot \delta(\vec{r}_0 - \vec{R}(t))\, d\vec{r}_0\, dt_0$

$= \theta \cdot \int \frac{1}{4\pi|\vec{r} - \vec{R}_0|} \cdot \delta\left(t - t_0 - \frac{|\vec{r} - \vec{R}|}{c}\right) q\, dt_0.$

yet $\vec{R}(t_0)$ has t_0. so we need to use the two approximation to eliminate it. $\Rightarrow \varphi = \frac{1}{4\pi r}\frac{e}{1 - \hat{n}\cdot\vec{v}/c} \approx \frac{e}{4\pi r}(1 + \hat{n}\cdot\vec{v}(t - r/c)/c)$. and $\vec{A} \ldots$

(c). from (b). we know φ and $\vec{A}$.

$\Rightarrow \vec{E} = -\nabla\varphi - \frac{1}{c}\partial_t \vec{A} = \frac{e}{4\pi r c^2}\underbrace{(-\vec{a} + \hat{n}(\hat{n}\cdot\vec{a}))}_{\vec{z}}$

P3.

☆ (d). $S = \frac{1}{\mu_0}(\cancel{\vec B\times\vec E}\ \vec E\times\vec B) = \frac{1}{\mu_0 c}[\vec E\times(\hat r\times\vec E)] = \frac{1}{\mu_0 c}[\hat r E^2 - \vec E(\hat r\cdot\vec E)]$

P461. $\frac{dP}{d\Omega} = \left(\frac{\vec r\cdot\vec u}{r c}\right)|S|\cdot r^2 = c\cdot|rE|^2 = \frac{e^2}{16\pi^2 c^3}a^2$

2015S EM2.

(a). $\vec A = \frac{\mu_0}{4\pi}\int\frac{\vec I(\vec r')}{r}dl'$ $\quad\vec B = \nabla\times\vec A = \frac{\mu_0}{4\pi}\int\frac{\vec I\times\vec r}{r^3}dl'$?

☆. solution is to use detailed Maxwell ~~eq~~ relation to solve the equation but I feel this is not a good way to do that somehow……

(b). use the result of circle current to calculate B. ✓.

(c) use $\nabla\times\vec E = -\partial_t\vec B$. or $\oint\vec E\cdot d\vec l = -\frac{d}{dt}\iint\vec B\cdot d\vec s$ $\quad\vec E = \vec E_\phi$.

(d). $\omega\to-\omega$. OK $\Rightarrow$ ω^2 correction to $\vec B$.

yet $\nabla\times\vec E = -\partial_t\vec B \Rightarrow \omega^3$ correction to $\vec E$.

far field happens while $\omega/c\cdot r > 1$. $\Rightarrow \sqrt{z^2+a^2} > \frac{c}{\omega}$ or $z > \frac{c}{\omega}$

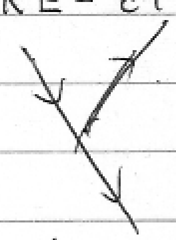
2015S. EM3.

(a). Maxwell equations:

$\vec J = \sigma\vec E \Rightarrow \nabla\times\vec B = (\sigma + \frac{1}{c}\partial_t)\vec E$. and $\nabla\times\vec E = \frac{1}{c}\partial_t\vec B$.

$\Rightarrow k^2 = i\cdot\frac{1}{c}\sigma\omega$. ✓

(b). $ikE = \frac{1}{c}(-i\omega)B$.

(c).  $\vec k\cdot\vec r = $ const or $E=0$. concern?

(d). find the boundary condition to determine H. standard.~

(e).

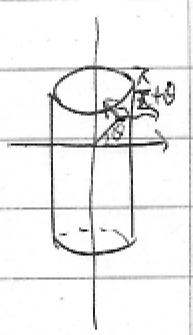
P4.

2015 F. EM1.

(a). think of $\vec{E}=0 \Rightarrow \vec{J} = \sigma_B \vec{B}$ and $\nabla\times\vec{B} = \vec{J}$ but this is violated by parity

(b). $\vec{E}_{\parallel}' = \vec{E}_{\parallel}$, $\vec{B}_{\parallel}' = \vec{B}_{\parallel}$, $\vec{E}_{\perp}' = \vec{E}_{\perp} + (\frac{\vec{u}}{c}\times\vec{B})_{\perp}$ $\quad \vec{B}_{\perp}' = \vec{B}_{\perp} - (\frac{\vec{u}}{c}\times\vec{E})_{\perp}$

$\vec{J}' = \sigma\vec{E}' \Rightarrow \vec{J} = -\rho\vec{u} + \vec{J}' = \vec{J}' = \sigma\vec{E}' = \sigma(\vec{E} + (\frac{\vec{u}}{c}\times\vec{B})_{\perp}) \approx \sigma(\vec{E} + \frac{\vec{u}}{c}\times\vec{B})$.



(c). $\vec{J} = \sigma(\vec{E} + \frac{\vec{u}}{c}\times\vec{B}) = \sigma\frac{uB}{c}\sin(\frac{\pi}{2}+\theta)(-\vec{e}_z) = \frac{\sigma\omega B}{c} r\cos\theta\,(-\vec{e}_z)$.

(d). $d\vec{f} = \vec{J}\times\vec{B} = \frac{\sigma\omega B^2}{c} r\cos\theta\,\vec{e}_x$

$d\vec{\tau} = \vec{r}\times d\vec{f} = \frac{\sigma\omega B^2}{c} y\cdot y(-\vec{e}_z)$.

$\Rightarrow \vec{\tau} = \int d\vec{\tau} = \frac{\sigma\omega B^2}{c}\cdot(-\vec{e}_z)\int y^2 dS \left(= \frac{1}{2}\int r^2 dS = \pi\cdot\int r^2 r\,dr = \pi\cdot\frac{R^4}{4}\right)$.

2015F EM2.

(a). $\nabla\cdot\vec{J} = \frac{1}{r}\partial_\phi J = \frac{1}{R}I_0\cos\phi\,\delta(\rho-R)\,\delta(z)$,

$\Rightarrow -i\omega\rho = -\nabla\cdot\vec{J} = \cdots \Rightarrow \rho = \frac{1}{i\omega}\nabla\cdot\vec{J}$

$\Rightarrow \vec{p} = \int \rho\,\vec{r}\,dV = \int \frac{1}{i\omega}\cdot\frac{1}{R}\partial_\phi J\,\vec{r}\,dV$.

$= \frac{1}{i\omega}\int \hat{r}\,I_0\cos\phi\,\delta(\rho-R)\,\rho\,d\rho\,d\phi$.

$= \frac{1}{i\omega}I_0 R\int \hat{r}\cos\phi\,d\phi$.

$= \frac{1}{i\omega}I_0 R\int(\cos^2\phi, \sin\phi\cos\phi, 0)\,d\phi$.

$= \frac{1}{i\omega}I_0 R\cdot(\pi, 0, 0)$.

(b). use retarded potential, we have:

$\vec{A} = e^{-i\omega t + ikr}/4\pi r \cdot \frac{-i\omega}{c}\vec{p}$.

$\vec{B} = \nabla\times\vec{A}$

$\frac{dP}{d\Omega} = \frac{c}{2} r^2 |B|^2 = \cdots$

(c). $\nabla\times\vec{E} = -\partial_t\vec{B} \propto i\vec{B} \propto i\vec{k}\times\vec{E} \Rightarrow \vec{k}\times\vec{B} \propto -\vec{E}$.

Langmuir equation

2015F EM3

(a). $\nabla^2 \Phi = -\rho/\varepsilon_0$, $\nabla \cdot \vec{J} + \partial_t \rho = 0$, $\frac{1}{2}mv^2 - e\Phi = \text{const}$.

$\Rightarrow \partial_x^2 \Phi = -\frac{J}{\varepsilon_0}\sqrt{\frac{m}{2e\Phi}}$.

(b). $\Rightarrow \Phi = A x^{\beta} \Rightarrow \beta = 4/3$. $A = \cdots$

as $\Phi(0) = 0$ $\Phi(D) = V$. ……

(c). ~~$J = \cdots$, $E = V/D \Rightarrow \sigma = J/E = J \cdot D/V = \cdots\cdots$ $R = \cdots$~~

$J \cdot S = I$. $R = V/I = V/J \cdot S$.