EM sheet. chuan cheng

P1. Maxwell:

$$\nabla\times\vec{E} = -\partial_t\vec{B} \qquad \nabla\times\vec{E} = -\partial_t\vec{B}$$
$$\nabla\cdot\vec{E} = \rho/\varepsilon_0 \quad \Leftrightarrow \quad \nabla\cdot\vec{D} = \rho_f$$
$$\nabla\cdot\vec{B} = 0 \qquad \nabla\cdot\vec{B} = 0$$
$$\frac{1}{\mu_0}\nabla\times\vec{B} = \vec{J} + \varepsilon_0\partial_t\vec{E} \qquad \nabla\times\vec{H} = \vec{J}_f + \partial_t\vec{D}$$

P2. boundary conditions (~~static~~)

$$\begin{cases}\vec{n}\times(\vec{E}_2-\vec{E}_1)=0 \\ \vec{n}\cdot(\vec{D}_2-\vec{D}_1)=\sigma_f\end{cases} \overset{\text{static}}{\Leftrightarrow} \begin{cases}(\varphi_1-\varphi_2)|_S = 0 \\ \varepsilon_2\partial_n\varphi_2 - \varepsilon_1\partial_n\varphi_1 = -\sigma_f\end{cases}$$

$$\begin{cases}\vec{n}\times(\vec{H}_2-\vec{H}_1)=\vec{K}_f \\ \vec{n}\cdot(\vec{B}_2-\vec{B}_1)=0\end{cases} \quad \rho_m!! \; ? \quad \begin{cases}(\varphi_{1m}-\varphi_{2m})|_S = \partial K_f \\ \mu_2\partial_n\varphi_{2m} - \mu_1\partial_n\varphi_{1m} = 0\end{cases}$$

P3. characteristic relations:

1. do we have source? $(\rho, \vec{J})$ or $(\rho_f, \vec{J}_f)$?
2. do we have material? $\begin{cases}\vec{D}=\varepsilon\vec{E}, \ \vec{D}=\varepsilon_0(\vec{E}+\vec{P}) \\ \vec{B}=\mu_0(\vec{H}+\vec{M}), \ \vec{B}=\mu\vec{H}\end{cases}$
3. do we have conductor? $\vec{J}=\sigma\vec{E}$ / $\vec{E}\equiv 0$.
4. compensation: $\vec{J}_m = \nabla\times\vec{M}$. $\rho_p = -\nabla\cdot\vec{P}$.
5. conservation of charge: $\partial_t\rho + \nabla\cdot\vec{J} = 0$

P4. def of potentials.

1. E-static. $\vec{E} = -\nabla\varphi$.
2. B-static $\vec{B} = \nabla\times\vec{A}$
3. dynamic i. Lorentzian $\nabla\cdot\vec{A} + \frac{1}{c}\partial_t\varphi = 0$.
   with. $\vec{E} = -\nabla\varphi - \frac{1}{c}\partial_t\vec{A}$. $\vec{B} = \nabla\times\vec{A}$. (new Maxwell!)
4. no source + static B.
   $\nabla\cdot\vec{H} = \rho_m/\mu_0$ and $\rho_m = -\mu_0\nabla\cdot\vec{M}$
   $H \triangleq -\nabla\varphi_m$
5. retarded potential
   $$\varphi = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\vec{r}', t-\frac{\tau}{c})}{\tau}dV'$$
   $$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}', t-\frac{\tau}{c})}{\tau}dV'. \quad (J=\rho V(\rho))$$

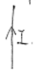
P5. Maxwell in integration:

$$\mathcal{E} = \oint\vec{E}\cdot d\vec{l} = -\partial_t\oiint\vec{B}\cdot d\vec{S}$$
$$\oint\vec{E}\cdot d\vec{S} = Q/\varepsilon_0.$$
$$\oint\vec{B}\cdot d\vec{S} = 0.$$
$$\oint\vec{B}\cdot d\vec{l} = \mu_0 I + \frac{1}{c^2}\partial_t\oiint\vec{E}\cdot d\vec{S}.$$

P6. Special Examples in Static Field.

1. $\vec{E} = \text{const} \Rightarrow \varphi = -Er\cos\theta$
2. 

   $W_{self} = Q^2/8\pi\varepsilon_0 a$
3. 

   $\varphi = -\frac{\lambda}{2\pi\varepsilon_0}\ln(R/R_0)$
4. 
   the ~~reverse~~ reflect points method
5. 

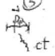
   $\vec{A} = -\frac{\mu_0 I}{2\pi}\ln\frac{R}{R_0}\vec{e}_z$
6. ? 

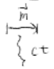
   $\varphi = \begin{cases}\frac{R_0^3}{3}\frac{\vec{M}_0\cdot\vec{R}}{R^3} & R>R_0 \\ \frac{1}{3}\vec{M}_0\cdot\vec{R} & R<R_0\end{cases}$
7. $U \overset{\text{dipole}}{=\!=} -\vec{m}\cdot\vec{B}_e = -\vec{p}\cdot\vec{E}$
   $U = \frac{1}{2}\int\rho V d\tau = \frac{1}{2}\int\vec{J}\cdot\vec{A}d\tau$
   $\vec{L} \overset{\text{dipole}}{=\!=} \partial_\theta U = \vec{m}\times\vec{B} = \vec{p}\times\vec{E}$.

P7. dipoles.

1. $f(\vec{x}) = f(\vec{x}_0) + (\Delta\vec{x}\cdot\nabla)f(\vec{x}) + \frac{1}{2!}(\Delta\vec{x}\cdot\nabla)^2 f + \cdots\cdots$
2. $\varphi_0 = \frac{Q}{4\pi\varepsilon R}$ $\quad\varphi_1 = \frac{\vec{p}\cdot\vec{R}}{4\pi\varepsilon_0 R^3}$ $\quad\varphi_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{6}\sum\mathcal{D}_{ij}\,\partial_i\partial_j\frac{1}{R}$
   where $\mathcal{D}_{ij} = \int 3\Delta x_i\Delta x_j\,\rho(x)dx'$
3. $\vec{A} = \frac{\mu_0}{4\pi R^3}\vec{m}\times\vec{R} = \frac{\mu_0}{4\pi}\int\frac{\vec{M}\times\vec{\tau}}{\tau^2}dV'$ $(m\sim S\cdot I)$
   $\vec{B} = \frac{\mu_0}{4\pi}\frac{1}{r^3}[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}]$ for pure dipole
4. ☆ $V = \frac{1}{4\pi\varepsilon_0}\int\frac{\vec{P}(\vec{r}')\cdot\hat{\tau}}{\tau^2}dV'$. $\quad\vec{P}\cdot dV' = \vec{p}$

   ☆ sphere $\vec{P}$: $V = \begin{cases}Pr\cos\theta/3\varepsilon_0 & r\le R \\ PR^3\cos\theta/3\varepsilon_0 r^2 & r>R\end{cases}$

   $\vec{E} = \begin{cases}-\frac{1}{3\varepsilon_0}\vec{P} \\ \frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{r^3}[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}]\end{cases}$
5. 

   $V = -\frac{p_0\omega}{4\pi\varepsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin[\omega(t-r/c)]$ $(\vec{p} = \vec{p}_0\cos\theta)$
   $\vec{A} = -\frac{\mu_0 p_0\omega}{4\pi r}\sin[\omega(t-r/c)]\hat{z}$ (where $I = \frac{dq}{dt} = \dot{p}_0/d$)
6. 

   $\vec{A} = -\frac{\mu_0 m_0\omega}{4\pi c}\left(\frac{\sin\theta}{r}\right)\sin[\omega(t-r/c)]\hat{\varphi}$
   $(I = I_0\cos\omega t,\ m = m_0(\cos\omega t)\hat{z},\ m_0 = \pi b^2 I_0)$

P8. E-M wave : $\vec{f} = \vec{f}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$

$\nabla = i\vec{k}$, $\partial_t = -i\omega$.

① on the surface : $\vec{k}\cdot\vec{x} = \vec{k}'\cdot\vec{x}'$, $k = \frac{\omega}{v}$

②. wave guide : $\left\{\begin{array}{l} \nabla\times\vec{E} = -\partial_t\vec{B} \quad \nabla\times\vec{B} = \frac{1}{c^2}\partial_t\vec{E} \\ \nabla = (\partial_x, \partial_y, ik) \end{array}\right.$

and $\nabla\cdot\vec{E} = 0$, $\nabla\cdot\vec{B} = 0$. only give me enough equations, leaving E_z, B_z undetermined (boundary)

P10. radiation (from P8 + P7)

①. $\vec{A}_{(1)} = \frac{\mu_0}{4\pi R}\cdot \cancel{e^{ikR}}\cdot\vec{\dot{p}}_t \Rightarrow \vec{B}_{(1)}, \vec{E}_{(1)}$

$\vec{S} = \vec{E}\times\vec{H} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{|\ddot{p}|^2}{32\pi^2\varepsilon_0 c^3 R^2}\sin^2\theta\,\vec{e}_R$

$\bar{S} = \frac{1}{2}\cdots$ $\quad \frac{dP}{d\Omega} = R^2|\vec{S}|\frac{\hat{r}\cdot\vec{u}}{rc}$; $P = \frac{1}{4\pi\varepsilon_0}\frac{|\ddot{p}|^2}{3c^3}$; $\frac{dP}{d\Omega} = \frac{|\vec{E}(\vec{p})|^2}{|\vec{E}_0|^2}$

②. basic procedure :

(1). write $\vec{A} \Rightarrow$ (2). get correct $\vec{B}$ and $\vec{E} \Rightarrow$

(3). get $\vec{S}$. $\Rightarrow$ (4) get P.

③ $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qc}{(rc - \vec{r}\cdot\vec{v})}$ $\quad \vec{A} = \frac{\vec{v}}{c^2}V(\vec{r}, t)$

$\vec{E} = \frac{q}{4\pi\varepsilon_0}\cdot\frac{r}{(\vec{r}\cdot\vec{u})^3}[(c^2 - v^2)\vec{u} + \vec{r}\times(\vec{u}\times\vec{a})]$ → decay as $|\vec{r}|^2 R^2$

$\vec{u} = c\hat{r} - \vec{v}$, $\vec{a} = \vec{\dot{v}}_t$

④ Some compensations

①. tensor : $\Theta^{\mu\nu}_{em} = F^{\mu\lambda}F_{\nu\lambda} + g^{\mu\nu}(-\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta})$.

have : $-\partial_\mu\Theta^{\mu\nu}_{em} = F^{\mu}{}_{\nu}\cdot J^\nu/c$

and : $\frac{dP^\mu}{d\tau} = eF^\mu{}_\nu\cdot U^\nu/c$ $\quad (P^\mu = mU^\mu)$

②. $d\vec{r} = dr\,\vec{e}_r + r d\theta\,\vec{e}_\theta + r\sin\theta d\varphi\,\vec{e}_\varphi$.

$\nabla = \vec{e}_r\partial_r + \vec{e}_\theta\frac{1}{r}\partial_\theta + \vec{e}_\varphi\frac{1}{r\sin\theta}\partial_\varphi$. $\quad (\alpha = r^2\sin\theta)$

$\nabla\cdot\vec{A} = \frac{1}{\alpha}[\partial_r(\frac{\alpha}{1}\cdot A_r) + \partial_\theta(\frac{\alpha}{r}A_\theta) + \partial_\varphi(\frac{\alpha}{r\sin\theta}A_\varphi)]$

③. $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$. $\quad \vec{f} = (\vec{E} + \vec{v}\times\vec{B})\rho = \rho\vec{E} + \vec{J}\times\vec{B}$

④. $\frac{dW}{dt} = \int_V \vec{F}\cdot\frac{d\vec{l}}{dt}dV = \int_V \vec{E}\cdot\vec{J}\,dV$

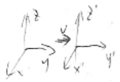
$= -\frac{d}{dt}\int_V \frac{1}{2}(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2)dV - \frac{1}{\mu_0}\oint_S(\vec{E}\times\vec{B})\cdot d\vec{S}$

$\Rightarrow \vec{f} = \nabla\cdot\overleftrightarrow{T} - \frac{1}{c^2}\partial_t\vec{S}$.

where $\overleftrightarrow{T} = \varepsilon_0\vec{E}\vec{E} + \frac{1}{\mu_0}\vec{B}\vec{B} - \frac{1}{2}\varepsilon_0 E^2\overleftrightarrow{I} - \frac{1}{2}\frac{1}{\mu_0}B^2\overleftrightarrow{I}$

P9. Relativity :

① $\beta = \frac{v}{c}$, $\gamma = (1-\beta^2)^{-1/2}$,



$\vec{x}^{\mu} = \Lambda^{\mu\nu}x_\nu'$

$\Lambda^{\mu\nu} = \begin{pmatrix} \gamma & \gamma\beta & & \\ +\gamma\beta & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$ ✓

②. $\frac{dt}{d\tau} = \gamma$, $\partial_\mu J^\mu = 0$. $\quad U^\mu = \frac{dx^\mu}{d\tau}$.

$\Box\vec{A} = -\mu_0 J_0$ $\quad \Box\varphi = -\rho/\varepsilon_0$

$F_{\mu\nu} \triangleq \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ & & & 0 \end{bmatrix}$

$\partial_\lambda F_{\mu\nu} = 0$. , $\partial_\nu F^{\mu\nu} = \mu_0 J^\mu$

③. $\vec{E}'_{\parallel} = \vec{E}_{\parallel}$, $\vec{B}'_{\parallel} = \vec{B}_{\parallel}$, $\vec{E}'_{\perp} = \gamma(\vec{E} + \vec{v}\times\vec{B})_{\perp}$.

$\vec{B}'_{\perp} = \gamma(\vec{B} - \frac{\vec{v}}{c^2}\times\vec{E})_{\perp}$ $\quad [F'_{\perp} = \frac{1}{\gamma}F_{\perp}, F'_{\parallel} = F_{\parallel}]$

④. procedure : (1) get relation in ① $\Rightarrow$

(2). write it clear the form ③ $\Rightarrow$

(3). write out the $x^\mu \to x'^\mu$ or $x'^\mu \to x^\mu$ in equation (2)

⑤. the power count down.

$P = \frac{\mu_0 q^2\gamma^6}{6\pi c}(a^2 - |\frac{\vec{v}\times\vec{a}}{c}|^2) \approx \frac{\mu_0 q^2}{6\pi c}a^2$

ass. $\vec{F}\cdot\vec{v} = -P = \frac{\mu_0 q^2}{6\pi c}a^2$

~~$\Rightarrow \vec{F} = m\vec{a}$~~

⑥ $\vec{E} = \frac{1}{4\pi\varepsilon_0}\gamma_0 q\frac{\vec{R}}{R^3}\cdot\frac{1}{(1-\beta^2\sin^2\theta)^{3/2}}$

⑦ $\vec{B}(z) = \frac{\mu_0 I}{2}\cdot\frac{R^2}{(R^2+z^2)^{3/2}}$

CHAPTER I

$= q\cdot\frac{\vec{r}}{4\pi\varepsilon_0 r^3} \Rightarrow \begin{cases} \oint \vec{E}\,d\vec{s} = \frac{Q}{\varepsilon_0} = \int_V \nabla\cdot\vec{E}\,dV = \int_V \rho/\varepsilon_0\,dV \Leftrightarrow \nabla\cdot\vec{E} = \rho/\varepsilon_0 \text{（动/静）} \\ \oint_l \vec{E}\cdot d\vec{l} = 0 \quad \int_S (\nabla\times\vec{E})\,d\vec{s} \Leftrightarrow \nabla\times\vec{E} = \vec{0} \text{（静）} \end{cases}$

$= \frac{dq}{dt} = \int_S \vec{J}\cdot d\vec{s} \qquad \vec{J} = \rho_i \vec{v_i}$

$= \int_V \nabla\cdot\vec{J}\,dV = \int_V -\frac{d\rho}{dt}\,dV \Rightarrow \nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$.（电荷守恒：动/静）

$\frac{L}{t}\int \rho\,dV = 0$.（边界条件） & $\nabla\cdot\vec{J} = 0$.（稳恒电流）

$\vec{F}_{12} = I_2 d\vec{l}_2 \times \vec{B}_1(b)$ $\quad \vec{B}_1 = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}_1(\vec{x}_1)\times(\vec{r}-\vec{x}_1)}{r^3}dV_1$（细致）. $\quad \vec{B} = \frac{\mu_0}{4\pi}\oint_L \frac{I\,d\vec{l}\times\vec{r}}{r^3}$（细导线上）

$\Rightarrow \oint_L \vec{B}\cdot d\vec{l} = \mu_0 I = \int_S (\nabla\times\vec{B})\cdot d\vec{s} = \mu_0\int_S \vec{J}\,d\vec{s} \Leftrightarrow \nabla\times\vec{B} = \mu_0\vec{J}$（静）

$\oint_S \vec{B}\cdot d\vec{s} = 0 = \int_V \nabla\cdot\vec{B}\,dV \Leftrightarrow \nabla\cdot\vec{B} = 0$（电流激发的，静/动）.

$\vec{B} = \nabla\times\frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(\vec{x}')dV'}{r}$（矢量运算，Poincare 定理）. $= \nabla\times\vec{A}$. $\qquad \nabla\cdot\vec{A} = 0$.（边值条件）

$\mathcal{E} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s} = \oint_L \vec{E}\cdot d\vec{l}$（电磁感应）. $\Leftrightarrow \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$（动/静）. ①

$\nabla\cdot(\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}) = 0$.（动/静）

$\nabla\times\vec{B} = \mu_0(\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t})$（动/静）. ②

①②③④ Maxwell 方程组.

$\nabla\cdot\vec{E} = \rho/\varepsilon_0$.（动/静） ③

$\nabla\cdot\vec{B} = 0$.（动/静） ④

~~$\vec{J}_f = \varepsilon\vec{E}$~~

$\begin{cases} \vec{J} = \sigma\vec{E} \text{（欧姆）} \\ \vec{D} = \varepsilon\vec{E} \\ \vec{B} = \mu\vec{H} \end{cases}$

$\int \rho_P\,dV = -\oint \vec{P}\cdot d\vec{s}$（边值条件）.

$\Updownarrow$

$\rho_P = -\nabla\cdot\vec{P}$.

$\sigma_P = -\vec{e}_n\cdot(\vec{P}_2 - \vec{P}_1)$.

$\longleftrightarrow \vec{P} = \chi_e\varepsilon_0\vec{E}$（各向同性介质）

$\xrightarrow{③} \nabla\cdot(\varepsilon_0\vec{E} + \vec{P}) = \rho_f \triangleq \nabla\cdot\vec{D}$

$\vec{D} = \varepsilon\vec{E}$.

$I_M = \oint n\vec{m}\cdot d\vec{l} = \int_S \vec{J}_M\,d\vec{s} = \int (\nabla\times\vec{M})\,d\vec{s}$

$\Leftrightarrow \vec{J}_M = \nabla\times\vec{M}$.（边值条件）.

$\vec{M} = \chi_M\vec{H}$（各向同性非铁磁）

$\frac{\partial\vec{P}}{\partial t} = \vec{J}_P$

$+②$

$\frac{1}{\mu_0}\nabla\times\vec{B} = \vec{J}_f + \vec{J}_M + \vec{J}_P + \varepsilon_0\frac{\partial\vec{E}}{\partial t} \Leftrightarrow \nabla\times(\frac{\vec{B}}{\mu_0} - \vec{M}) = \vec{J}_f + \frac{\partial\vec{D}}{\partial t} \triangleq \nabla\times\vec{H}$

$\vec{B} = \mu\vec{H}$

Chapter 2. 静电场

$$\begin{cases}\nabla\times\vec E=0\\ \nabla\cdot\vec D=\rho_f\end{cases}\quad(静)$$

$\downarrow$ $\vec D=\varepsilon\vec E$（各向同性+线性介质）

$$\Delta\varphi=-\frac{\rho}{\varepsilon}\quad(\vec E=-\nabla\varphi)$$

$\downarrow$ 边 $\begin{cases}\vec e_n\times(\vec E_2-\vec E_1)=0 & 切向相等不突变.\\ \vec e_n\cdot(\vec D_2-\vec D_1)=\sigma_f & 纵向突变.\end{cases}$

$\mathcal{T}_1$ 介质边界条件

$$\begin{cases}(\varphi_1-\varphi_2)_S=0\\ \varepsilon_2\frac{\partial\varphi_2}{\partial n}-\varepsilon_1\frac{\partial\varphi_1}{\partial n}=-\sigma_{自由}\\ (可推:\ \sigma_p=-\vec e_n\cdot(\vec P_2-\vec P_1)=\varepsilon_0\vec e_n\cdot(\vec E_1-\vec E_2)-\sigma_f)\end{cases}$$

隐含条件 $\begin{cases}\varphi|_{R=0} & 有界\\ \varphi|_{R\to\infty} & 有模型\end{cases}$

$\mathcal{T}_2$ 导体边界条件： $\begin{cases}内部无净电荷\\ 内部无电场线\\ 外表电场沿法向\end{cases}\Longleftrightarrow\begin{cases}\varphi=\text{const}\\ \varepsilon\frac{\partial\varphi}{\partial n}=-\sigma_f & 自由电荷(非极化)\\ \oint-\varepsilon\frac{\partial\varphi}{\partial n}dS=Q_f & 导体电荷\end{cases}$

$\mathcal{T}_3$ 能量 $W=\frac12\int_\infty(\vec E\cdot\vec D)dV$

$=\frac12\int_V\rho\varphi\,dV$

$\downarrow$（导体·电容）

$W=\frac12 Q\cdot U$

$\mathcal{T}_5$ Green函数方法

1. 无界空间： $\varphi(x)=\frac{1}{4\pi\varepsilon_0}\frac{1}{|\vec r-\vec r'|}$
2. 上半空间： $\varphi(x)=\frac{1}{4\pi\varepsilon_0}\left[\frac{1}{|\vec r-(x',y',z')|}-\frac{1}{|\vec r-(x',y',-z')|}\right]$
3. 球外空间： $\varphi(x)=\frac{1}{4\pi\varepsilon_0}\left[\frac{1}{|\vec r-(x',y',z')|}-\frac{1}{|\vec r-镜像|}\right]$

$\varphi(\vec x)=\int_V G(\vec x',\vec x)\rho(\vec x')dV'-\varepsilon_0\oint_S\varphi(\vec x')\frac{\partial}{\partial n'}G(\vec x',\vec x)dS'$ （第一类）

$\mathcal{T}_6$ 电多极矩

$$f(\vec x-\vec x')=f(\vec x)-\sum x_i'\frac{\partial}{\partial x_i}f(\vec x)+\frac{1}{2!}\sum x_i'x_j'\frac{\partial^2}{\partial x_i\partial x_j}f(\vec x)+\cdots$$

$$=f(\vec x)-\vec x'\cdot\nabla f(x)+\frac{1}{2!}(\vec x'\cdot\nabla)^2f(\vec x)$$

$$\begin{cases}\varphi_0=\frac{Q}{4\pi\varepsilon_0R} & \varphi_1=\frac{\vec p\cdot\vec R}{4\pi\varepsilon_0R^3}\\ \varphi_2=\frac{1}{4\pi\varepsilon_0}\frac16\sum\mathscr{D}_{ij}\frac{\partial^2}{\partial x_i\partial x_j}\frac1R & \mathscr{D}_{ij}=\int_V3x_i'x_j'\rho(\vec x')dV'\end{cases}$$

$$\begin{cases}Q=\int_V\rho(\vec x')dV' & \vec p=\int_V\rho(\vec x')\vec x'dV'\\ \mathscr{D}_{ij}=\int_V3x_i'x_j'\rho(\vec x')dV'\end{cases}$$

$\mathcal{T}_4$ eg.

~~导体的阻断效果~~

6. 电偶极子电势 $\varphi=\frac{\vec p\cdot\vec R}{4\pi\varepsilon R^3}$
1. 匀强场 $\varphi=-\vec E\cdot\vec x=-Er\cos\theta$
2. 导体球能量 $W=\frac{Q^2}{8\pi\varepsilon_0a}$

※3. 直导线电势 $\varphi=-\frac{\tau}{2\pi\varepsilon_0}\ln\frac{R}{R_0}$

4. 平板、球的镜像
5. 电容率 ε 于匀强场中电势

方法：分离使 $\Delta\varphi=0$ 的特殊项，然后求解之。注意电势零点和源零点，有时需拉出来。导体有阻断的效果。

Chapter 3. 静磁场

§1.
$$\begin{cases} \nabla\times\vec{H} = \vec{J}_f \\ \nabla\cdot\vec{B} = 0 \end{cases} \Rightarrow \begin{cases} \vec{e}_n\times(\vec{H}_2-\vec{H}_1) = \vec{\alpha}_f \\ \vec{e}_n\cdot(\vec{B}_2-\vec{B}_1) = 0 \end{cases}$$

$$\Downarrow \; \vec{B} = \nabla\times\vec{A}$$

$$\begin{cases} \nabla(\nabla\cdot\vec{A}) - \Delta\vec{A} = \mu\vec{J} \\ W = \frac{1}{2}\int \vec{B}\cdot\vec{H}\,dV = \cancel{\int \vec{A}\cdot\vec{J}\,dV} \cdot \frac{1}{2}\int \vec{J}\cdot\vec{A}\,dV \quad (*) \end{cases}$$

§2.
$$\begin{cases} \nabla\times\vec{H} = 0 \\ \nabla\cdot\vec{B} = 0 \\ \vec{B} = \mu_0(\vec{H}+\vec{M}) \end{cases} \xrightarrow{\rho_m = -\mu_0\nabla\cdot\vec{M}} \begin{cases} \nabla\times\vec{H} = 0 \\ \nabla\cdot\vec{H} = \rho_m/\mu_0 \\ \rho_m = -\mu_0\nabla\cdot\vec{M} \\ \vec{H} = -\nabla\varphi_m \\ \vec{B} = \mu_0(\vec{H}+\vec{M}) \\ \Delta\varphi_m = -\rho_m/\mu_0 \end{cases}$$

(**) - 不可滥用！这是个特别的手段，是类比电场所得：

$$\Rightarrow \begin{cases} \nabla\cdot\vec{H} = \rho_m/\mu_0 \\ \rho_m = -\mu_0\nabla\cdot\vec{M} \\ \nabla\times\vec{H} = 0 \Leftrightarrow \vec{H} = -\nabla\varphi_m \\ \begin{cases} \vec{e}_n\cdot(\vec{B}_2-\vec{B}_1) = 0 \\ \vec{e}_n\times(\vec{H}_2-\vec{H}_1) = 0 \end{cases} \Leftrightarrow \begin{cases} \mu_2\frac{\partial\varphi_{2m}}{\partial n} - \mu_1\frac{\partial\varphi_{1m}}{\partial n} = 0 \\ \varphi_{2m} = \varphi_{1m}|_S \end{cases} \end{cases}$$

§3.
$$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{J}'dV'}{r}$$
$$= \frac{\mu_0}{4\pi}\int\vec{J}'\left[\frac{1}{R} - \vec{x}'\cdot\nabla\frac{1}{R} + \cdots\right]dV'$$
$$= 0 - \frac{\mu_0 I}{4\pi}\oint\vec{x}'\cdot\frac{\vec{R}}{R^3}d\vec{l}' + \cdots\cdots$$
$$= \frac{\mu_0}{4\pi R^3}\vec{m}\times\vec{R} \quad \left(\vec{m} = \frac{I}{2}\oint\vec{x}'\times d\vec{l}'\right) ☆$$

磁通 $\Phi = \oint_L \vec{A}\cdot d\vec{l}$

☆ 外磁场中势能. $U = -\vec{m}\cdot\vec{B}_e$
$$\vec{F} = -\nabla U = \vec{m}\cdot\nabla\vec{B}_e$$
$$\vec{L} = -\frac{\partial U}{\partial\theta} = \vec{m}\times\vec{B}_e$$

1. 无穷长直导线. $\vec{A} = -\frac{\mu_0 I}{2\pi}\ln\frac{R}{R_0}\vec{e}_z$

2. $\vec{M}_0$ 下均匀磁化铁球磁场.
$$\varphi = \begin{cases} \frac{R_0^3}{3}\frac{\vec{M}_0\cdot\vec{R}}{R^3} & R > R_0 \\ \frac{1}{3}\vec{M}_0\cdot\vec{R} & R < R_0 \end{cases}$$

3. 磁偶极矩磁场 $\vec{B}^{(1)} = -\frac{\mu_0}{4\pi}(\vec{m}\cdot\nabla)\frac{\vec{R}}{R^3}$
$$\varphi_m^{(1)} = \frac{\vec{m}\cdot\vec{R}}{4\pi R^3}$$

☆4. 磁偶极子在外场中势函 $U = -\vec{m}\cdot\vec{B}_e$

Chapter 4. 电磁波传播.

α1.
$$\begin{cases}\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}\\ \nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t}\\ \nabla\cdot\vec{D}=0\\ \nabla\cdot\vec{B}=0\end{cases} \Rightarrow \begin{cases}\Delta\vec{E} - \mu\varepsilon\ddot{\vec{E}}=0\\ \Delta\vec{B} - \mu\varepsilon\ddot{\vec{B}}=0\end{cases}$$

$\Downarrow$ 消去时间因子. 不改变记号.

α2.
$$\begin{cases}\vec{E}(\vec{x},t) = \vec{E}\cdot e^{-i\omega t}\\ \vec{B}(\vec{x},t) = \vec{B}\cdot e^{-i\omega t}\\ \nabla\times\vec{E} = i\omega\mu\vec{H}\\ \nabla\times\vec{H} = i\omega\varepsilon\vec{E}\\ \nabla\cdot\vec{E}=0\\ \nabla\cdot\vec{H}=0\end{cases} \Rightarrow \begin{cases}\Delta\vec{E}+\omega^2\mu\varepsilon\vec{E}=0\\ \Delta\vec{H}+\omega^2\mu\varepsilon\vec{H}=0\end{cases} \Rightarrow \begin{cases}\Delta\vec{E}+\omega^2\mu\varepsilon\vec{E}=0\\ \nabla\cdot\vec{E}=0\\ \vec{B}=-\frac{i}{\omega}\nabla\times\vec{E}\end{cases}$$

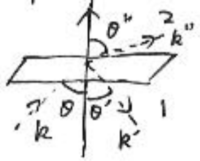
α3. $v=\frac{\omega}{k}=\frac{1}{\sqrt{\mu\varepsilon}}\triangleq\frac{c}{n}$ 平面电磁波

$\nabla = i\vec{k}$

$\vec{S}=\vec{E}\times\vec{H}=vW\vec{e}_k$

$\overline{\vec{S}}=\frac{1}{2}\mathrm{Re}(\vec{E}^*\times\vec{H})=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}E_0^2\vec{e}_k$

α4. 介质界面.



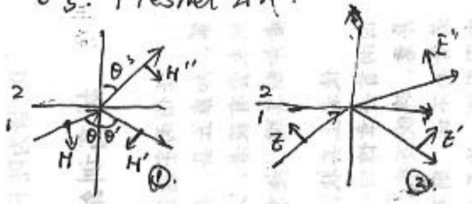
$$\begin{cases}\theta=\theta'\\ k=k'=\frac{\omega}{v_1}\end{cases} \quad \begin{cases}\frac{\sin\theta}{\sin\theta''}=\frac{v_1}{v_2}\\ k''=\frac{\omega}{v_2}\end{cases}$$

$\vec{k}\cdot\vec{x}=\vec{k}'\cdot\vec{x}=\vec{k}''\cdot\vec{x}$ ($\vec{x}$在平面内)

边界条件是: $\Sigma\varepsilon E^{\perp}=\text{const}$ $\Sigma B^{\perp}=\text{const}$

$\Sigma E^{\parallel}=\text{const}$ $\Sigma\frac{1}{\mu}B^{\parallel}=\text{const}$.

α5. Fresnel公式.



① $\vec{E}\perp$入射面. (s偏)

$$\begin{cases}E+E'=E''\\ H\cos\theta-H'\cos\theta'=H''\cos\theta''\\ H=\sqrt{\frac{\varepsilon}{\mu}}E \quad (\mu\approx\mu_0)\end{cases}$$

② $\vec{E}\parallel$入射面. (p偏)

$$\begin{cases}E\cos\theta-E'\cos\theta'=E''\cos\theta''\\ H+H'=H''\\ H=\sqrt{\frac{\varepsilon}{\mu}}E \quad (\mu\approx\mu_0)\end{cases}$$

③ 全反射下.

$$\begin{cases}\vec{E}''=\vec{E}_0''e^{-\kappa z}e^{i(k_x''x-\omega t)}\\ \kappa=k\sqrt{\sin^2\theta-n_{21}^2}\end{cases}$$

α6. 导体内传播. ($J\neq0$).

$$\begin{cases}\frac{\sigma}{\varepsilon\omega}>>1\\ \nabla\times\vec{E}=i\omega\mu\vec{H}\\ \nabla\times\vec{H}=-i\omega\varepsilon\vec{E}+\sigma\vec{E}\\ \nabla\cdot\vec{E}=0\\ \nabla\cdot\vec{H}=0\end{cases}$$

~~传导~~ 位移 << 传导

同上结论.

α7. 波导与谐振腔.

Chapter 5. 电磁辐射

§1.

$$\begin{cases} \nabla\cdot\vec{D}=\rho \\ \nabla\cdot\vec{B}=0 \\ \nabla\times\vec{E}=-\vec{B}_t \\ \nabla\times\vec{H}=\vec{J}+\vec{D}_t \end{cases} \Leftrightarrow \begin{cases} \vec{B}=\nabla\times\vec{A} \\ \vec{E}=-\vec{A}_t-\nabla\varphi \\ \varepsilon\nabla(-\vec{A}_t-\nabla\varphi)=\rho \\ \nabla\times(\nabla\times\vec{A})=\mu\vec{J}+\varepsilon\mu(-\vec{A}_{tt}-\nabla\varphi_t) \end{cases}$$

§2. 库仑条件

$$\begin{cases} \nabla\cdot\vec{A}=0 \\ \varepsilon\Delta\varphi=-\rho \\ -\Delta\vec{A}=\mu\vec{J}+\varepsilon\mu(-\vec{A}_{tt}-\nabla\varphi_t) \end{cases}$$

洛伦兹条件

$$\begin{cases} \nabla\cdot\vec{A}+\frac{1}{c^2}\varphi_t=0 \\ \varepsilon(\Delta\varphi-\frac{1}{c^2}\varphi_{tt})=\rho \\ \nabla\times(\nabla\times\vec{A})=\mu\vec{J}+\varepsilon\mu(-\vec{A}_{tt}+c^2\nabla(\nabla\cdot\vec{A})) \\ \Leftrightarrow -\Delta\vec{A}=\mu\vec{J}+\frac{1}{c^2}(-\vec{A}_{tt}) \end{cases}$$

伦敦规范：$\nabla\cdot\vec{A}=0$. $\vec{e}_n\cdot\vec{A}|_S=0$.（超导）

§3. 平面电磁波

$$\begin{cases} \vec{A}=\vec{A}_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)} \\ \varphi=\varphi_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)}=\frac{c^2}{\omega}\vec{k}\cdot\vec{A} \\ \varphi_0=\frac{c^2}{\omega}\vec{k}\cdot\vec{A}_0 \\ \nabla=i\vec{k} \quad \frac{\partial}{\partial t}=-i\omega \end{cases} \Rightarrow \begin{cases} \vec{B}=i\vec{k}\times\vec{A} \\ \vec{E}=i\omega\vec{A}-i\vec{k}\varphi \end{cases}$$

$$=i\omega\vec{A}-i\vec{k}\frac{c^2}{\omega}(\vec{k}\cdot\vec{A})$$

$$\overset{①}{=} ic(k\vec{A}-(\vec{k}\cdot\vec{A})\vec{e}_k)$$

$$\overset{②}{=} -c\vec{e}_k\times\vec{B}$$

§4. 推迟势（d'alambert）

方程 $\Delta\varphi-\frac{1}{c^2}\varphi_{tt}=-\rho/\varepsilon_0$ 的1个试解为 $\varphi=\frac{Q(t-\frac{r}{c})}{4\pi\varepsilon_0 r}$（分离变量型！）

其中 $\rho=Q\cdot\delta(x)$. $\Rightarrow \vec{A}=\frac{\mu_0}{4\pi}\int_V\frac{J(\vec{x}',t-\frac{r}{c})}{r}dV'$

满足洛氏条件！

§5. 电偶极辐射

§4中令 $\vec{J}=\vec{J}(\vec{x}')e^{-i\omega t}$

$$\Rightarrow \begin{cases} \vec{A}=\vec{A}(x)e^{-i\omega t} \\ \vec{A}(x)=\frac{\mu_0}{4\pi}\int_V\frac{\vec{J}(\vec{x}')e^{ikr}}{r}dV' \end{cases}$$

$$\begin{cases} r\ll\lambda & \text{近区} \\ r\sim\lambda & \text{感应区} \\ r\gg\lambda & \text{远区} \end{cases}$$

☆. 对 $r\approx R-\vec{e}_R\cdot\vec{x}'$ 作展开

$$\vec{A}(x)\approx\frac{\mu_0}{4\pi R}e^{ikR}\int_V J(\vec{x}')(1-ik\vec{e}_R\cdot\vec{x}'+\cdots)dV'$$

注意到 $\frac{d\vec{p}}{dt}=\frac{d}{dt}(Q\vec{l})=I\vec{l}=\int_V\vec{J}dV$

取第一项 $A_{(1)}=\frac{\mu_0}{4\pi R}e^{ikR}\dot{\vec{p}}$

☆.
$$\begin{cases} B_{(1)}\approx ik\vec{e}_R\times A_{(1)}=\frac{1}{4\pi\varepsilon_0c^3R}e^{ikR}\ddot{\vec{p}}\times\vec{e}_R \\ E_{(1)}\approx c\vec{B}\times\vec{e}_R=\frac{e^{ikR}}{4\pi\varepsilon_0c^3R}(\ddot{\vec{p}}\times\vec{e}_R)\times\vec{e}_R \end{cases}$$

$$\Leftrightarrow \begin{cases} \nabla\approx ik\vec{e}_R \\ \frac{\partial}{\partial t}\approx -i\omega \end{cases}$$

$$\overline{\vec{S}}=\frac{1}{2}\mathrm{Re}(\vec{E}^*\times\vec{H})=\frac{|\ddot{\vec{p}}|^2}{32\pi^2\varepsilon_0c^3R^2}\sin^2\theta\,\vec{e}_R$$

☆ $P=\oint|\vec{S}|R^2d\Omega=\frac{1}{4\pi\varepsilon_0}\frac{|\ddot{\vec{p}}|^2}{3c^3}$

实例：短天线：$I(z)=I_0(1-\frac{2}{l}|z|)$.

Chapter 6. 狭义相对论.

§1. 相对性原理 + 光速不变原理 + 线性变换 + 空间无特定方向.

↓

间隔不变性 $S^2=c^2t^2-x^2=\text{const}$.

↓正交变换

☆. $(x,y,z,t) \longleftrightarrow (\gamma(x-vt), y, z, \gamma(t-\frac{v}{c^2}x))$. 洛伦兹公式.

§2. 时空理论: $S^2=0$. 光波联系

$S^2>0$. 类时间隔. → 绝对的过去和未来

$S^2<0$. 类空间隔. → 相对的过去和未来.

事件理论 → 洛伦兹变换.

钟慢与尺缩: 观察者位于V系. 对象物体静止于V'系.

前提条件: 线性变换. 有调整钟的要求. 即 $(0,0)\longleftrightarrow(0,0)$

速度变换: $u_x=\frac{dx}{dt}=\frac{u_x'+v}{1+\frac{vu_x'}{c^2}}$ $\quad u_y=\frac{u_y'\sqrt{1-v^2/c^2}}{1+vu_x'/c^2}$ $\quad u_z=\frac{u_z'\sqrt{1-v^2/c^2}}{1+vu_x'/c^2}$

特别: 介质中光速恒为 $\frac{c}{n}$. 但不影响上述的c.

注意 P45. 光在不同参考系所行走路径并不固定的!

§3. 四维.

(1) 洛氏变换特征矩阵为 $\tilde{a}=\begin{pmatrix}\gamma & & & i\beta\gamma\\ & 1 & & \\ & & 1 & \\ -i\beta\gamma & & & \gamma\end{pmatrix}$ 四维为 (x,y,z,ict).

形式: $x_\mu'=a_{\mu\nu}x_\nu$ $\quad \tilde{a}a=I$. 为四维矢量.

(2). 固有时 $d\tau=\frac{1}{c}ds$. $\quad ds^2=-dx_\mu dx_\mu$.

(3). $U_\mu=\frac{dx_\mu}{d\tau}=\gamma(u_x,u_y,u_z,ic)$ ∝ x_μ 为四维矢量.

(4). $\vec{k}\cdot\vec{x}-\omega t=k_\mu\cdot x_\mu=\text{const}$. ⇒ k_μ 为协变量. 故为四维矢量.

$k_\mu=(k_x,k_y,k_z,i\frac{\omega}{c})$.

(5). $J_\mu=\rho_0U_\mu=(J_x,J_y,J_z,ic\rho)$. ∝ U_μ ∝ x_μ. 为4维矢量.

由于 $\frac{\partial J_\mu}{\partial x_\mu}=\nabla\cdot\vec{J}+\rho_t=0$. 故 J_μ 协变. 故为4维矢量.

(6). $\Box=\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}=\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x_\mu}$ $\quad A_\mu=(A_x,A_y,A_z,ic\varphi)$.

有 $\Box A_\mu=-\mu_0J_\mu$ 协变性. 故为4维矢量.

$\frac{\partial A_\mu}{\partial x_\mu}=0$

☆ (7). 电磁场张量.

$$F_{\mu\nu}=\begin{bmatrix}0 & B_3 & -B_2 & -\frac{i}{c}E_1\\ -B_3 & 0 & B_1 & -\frac{i}{c}E_2\\ B_2 & -B_1 & 0 & -\frac{i}{c}E_3\\ \frac{i}{c}E_1 & \frac{i}{c}E_2 & \frac{i}{c}E_3 & 0\end{bmatrix}=\frac{\partial A_\nu}{\partial x_\mu}-\frac{\partial A_\mu}{\partial x_\nu}$$

$F_{\mu\nu}'=a_{\mu\lambda}a_{\nu\tau}F_{\lambda\tau}$. ①. $\quad \frac{\partial F_{\mu\nu}}{\partial x_\nu}=\mu_0J_\mu$ (对重复下标求和) ②.

$\frac{\partial F_{\mu\nu}}{\partial x_\lambda}+\frac{\partial F_{\nu\lambda}}{\partial x_\mu}+\frac{\partial F_{\lambda\mu}}{\partial x_\nu}=0$. ③ $\quad$ ②+③代表 Maxwell 方程.

(8). 电磁场变换:

$E_1'=E_1$, $B_1'=B_1$. $E_2'=\gamma(E_2-vB_3)$ $\quad B_2'=\gamma(B_2+\frac{v}{c^2}E_3)$.

$E_3'=\gamma(E_3+vB_2)$ $\quad B_3'=\gamma(B_3-\frac{v}{c^2}E_2)$

即 $\vec{E}_\parallel'=\vec{E}_\parallel$ $\quad \vec{B}_\parallel'=\vec{B}_\parallel$ $\quad \vec{E}_\perp'=\gamma(\vec{E}+\vec{v}\times\vec{B})_\perp$ $\quad \vec{B}_\perp'=\gamma(\vec{B}-\frac{\vec{v}}{c^2}\times\vec{E})_\perp$

(9). 不变量.

$\frac{1}{2}F_{\mu\nu}F_{\mu\nu}=B^2-\frac{1}{c^2}E^2$

$\frac{i}{8}\varepsilon_{\mu\nu\lambda\tau}F_{\mu\nu}F_{\lambda\tau}=\frac{i}{c}\vec{B}\cdot\vec{E}$.

§(10). 相对论力学.

$K_\mu=\frac{dp_\mu}{d\tau}$

$K=(\gamma\vec{F},\frac{i}{c}\gamma\vec{F}\cdot\vec{v})$.

$p_\mu=(\vec{p},\frac{i}{c}W)$.

$p_\mu p_\mu=-m_0^2c^2=\vec{p}^2-\frac{W^2}{c^2}$.

$W=mc^2=\gamma m_0c^2$

$\vec{p}=\gamma m_0\vec{v}=m\vec{v}$