Weak *P*- and *PT*-odd effects in electric conductors

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(Submitted 21 May 1986; resubmitted 20 August 1986)
Zh. Eksp. Teor. Fiz. 92, 589–593 (February 1987)

It is shown that *P*- and *PT*-odd currents, due to weak interaction between the conduction electrons and the lattice nuclei, can flow in electric conductors in a constant magnetic field. The Boltzmann kinetic equation is used, which allows also the concomitant Hall effect to be considered. The conduction-electron and the lattice-nuclei spins are assumed to be polarized and not aligned, after the external field is turned on, with the resultant magnetic field during the time of the magnetic relaxation. The conditions that permit a longer observation time for the expected effect are discussed.

1. The present article is a continuation of Ref. 1, which deals with weak spatial-parity violating interaction between conduction electrons in metals and nuclei of crystal-lattice atoms. According to Ref. 1 this interaction can result in electric-current flow in conductors in a stationary magnetic field:

$$\mathbf{i}=\sigma_{\mathscr{H}}\left[\frac{1}{\hbar}\langle\mathbf{s}\rangle,\langle\vec{\mathscr{H}}\rangle\right], \tag{1}$$

where **i** is the current density, $\langle\mathbf{s}\rangle$ is the average electron spin, $\langle\vec{\mathscr{H}}\rangle$ is the average magnetic field acting on the electron, and $\sigma_{\mathscr{H}}$ is a proportionality coefficient that plays the role of the electric conductivity.

In the present paper, in contrast to Ref. 1, we consider, first, the effective-Hamiltonian *P*- and *PT*-odd terms capable of causing current flow, and discuss the possibility of separate observation of the corresponding effects. Second, we consider here a new a possible experimental verification of the effects in question. In Ref. 1, the experimental situation in ferromagnets was in certain respects incorrectly understood. It can be seen from (1) that for current to exist it is necessary that the vector $\langle\mathbf{s}\rangle$ (i.e., the magnetization) not have the same direction as the vector $\langle\vec{\mathscr{H}}\rangle$ of the combined internal and external magnetic fields acting on the electron in the semiconductor. The ferromagnet considered in Ref. 1 had therefore high coercivity, so that its magnetic-relaxation time (i.e., the time for the magnetization vector to align itself with the total field) was assumed long compared with the measurement time. Actually, however, for ferromagnets, including those with high coercivity, the magnetic-relaxation time is approximately the same as for paramagnets, and this imposes additional requirements on the experiments discussed in Ref. 1.[1)] In our opinion, nevertheless, experiments of this kind are possible.

2. In weak electron interaction between an electron and nuclei in a lattice, the effective Hamiltonian containing *P*- and *PT*-odd terms can be written according to Refs. 2 and 3 in the form

$$\hat{H}_W=\hat{H}_W{}^P+\hat{H}^{PT}, \tag{2}$$

$$\hat{H}_W{}^P=\frac{G\hbar^2}{\sqrt{2}\,mc^2}\Big\{(Zg_1\mathbf{s}+g_2\mathbf{I})\Big[\hat{\mathbf{p}},\sum_{\mathbf{a}}\delta(\mathbf{r}-\mathbf{a})\Big]_+ +ig_3[\mathbf{s}\mathbf{I}]\Big[\hat{\mathbf{p}},\sum_{\mathbf{a}}\delta(\mathbf{r}-\mathbf{a})\Big]_-\Big\}, \tag{3}$$

$$\hat{H}_W{}^{PT}=\frac{G\hbar^2}{\sqrt{2}\,mc^2}\Big\{i(Zh_1\mathbf{s}+h_2\mathbf{I})\Big[\hat{\mathbf{p}},\sum_{\mathbf{a}}\delta(\mathbf{r}-\mathbf{a})\Big]_- +h_3[\mathbf{s}\mathbf{I}]\Big[\hat{\mathbf{p}},\sum_{\mathbf{a}}\delta(\mathbf{r}-\mathbf{a})\Big]_+\Big\}. \tag{4}$$

Here m, **s**, and **r** are the mass, spin, and coordinate of the electron, $\hat{\mathbf{p}}=-i\hbar\nabla$ is the electron momentum operator, Z, **I**, and **a** are the charge, spin, and coordinate of the nucleus, G is the Fermi constant, g_i and h_i are the constants of the *P*- and *PT*-odd weak interactions, and $[\ldots]_\mp$ denotes a commutator (anticommutator). The summation in (3) and (4) is over all the nuclei in the crystal.

In the presence of an external electric field, in view of the gauge invariance, the operator $\hat{\mathbf{p}}$ in (5) must be replaced by $\mathbf{p}+e\mathbf{A}/c$.

The Hamiltonian (2) describes the contact interaction of the electron with the lattice nuclei. It is more convenient to replace it by an effective interaction that is "smeared out" over the crystal and is written in the compact form

$$\hat{H}_W=\frac{1}{m}\mathbf{X}\left(\hat{\mathbf{p}}+\frac{e}{c}\mathbf{A}\right), \tag{5}$$

$$\mathbf{X}=Q_1{}^P\mathbf{s}+Q_2{}^P\mathbf{I}+Q_3{}^{PT}[\mathbf{s}\mathbf{I}]. \tag{6}$$

The constants Q_i are determined from the condition

$$\langle\Psi|\hat{H}_W|\Psi\rangle=\langle\Psi|H_W|\Psi\rangle, \tag{7}$$

where Ψ is the wave function of the conduction electron in the crystal. We note that with such a smearing of the interaction the nuclear spin can also be regarded as distributed over the crystal and regarded formally as an additional degree of freedom for the electron.

3. The total nonrelativistic Hamiltonian for a conduction electron in a crystal can be written in the general case, at the required accuracy in the form

$$\hat{H}=\frac{1}{2m}\hat{\mathbf{P}}^2-eV+H_s+H_I, \tag{8}$$

$$\hat{\mathbf{P}}=\hat{\mathbf{p}}+\frac{e}{c}\mathbf{A}+\mathbf{X}, \tag{9}$$

$$H_s=\frac{e}{mc}\vec{\mathscr{H}}\mathbf{s}=\mu_s\vec{\mathscr{H}}\mathbf{s}, \tag{10}$$

$$H_I=\mu_N g\vec{\mathscr{H}}\mathbf{I}, \tag{11}$$

where V is the resultant electrostatic potential acting on the electron, $\mathscr{H}$ the resultant magnetic field strength, μ_N the

0038-5646/87/020333-03$04.00

nuclear magneton, and g the nuclear g-factor. We have aded here to the electron Hamiltonian the interaction between the smeared nuclear spin and the magnetic field.

To calculate the electron-velocity operator v and the force F acting on the electron, we use the equations

$$\hat{\mathbf{v}}=\dot{\mathbf{r}}=\frac{i}{\hbar}[\hat{H},\mathbf{r}]_-=\frac{1}{m}\hat{\mathbf{P}}, \tag{12}$$

$$\hat{\mathbf{F}}=m\ddot{\mathbf{r}}=\frac{i}{\hbar}[\hat{H},\hat{\mathbf{P}}]_-. \tag{13}$$

Calculation of the commutators in (13) yields

$$\hat{\mathbf{F}}=-e\mathbf{E}+\mu_s[\vec{\mathscr{H}}\hat{\mathbf{P}}]+\mu_s Q_1^P[\vec{\mathscr{H}}\mathbf{s}]+\mu_N g Q_2^P[\vec{\mathscr{H}}\mathbf{I}] + \mu_s Q_3^{PT}[\mathbf{I}[\mathbf{s}\vec{\mathscr{H}}]]-\mu_N g Q_3^{PT}[\mathbf{s}[\mathbf{I}\vec{\mathscr{H}}]]. \tag{14}$$

The first term here is the usual electrostatic force, E is the external-electric-field strength, the second term is the Lorentz force, and the remaining terms are the P- and PT-odd forces that result from the weak interaction.

4. Substituting the force (14) in the Boltzmann kinetic equation, we obtain an expression for the current (see the analogous derivation in Refs. 1 and 4)

$$\mathbf{i}=\mathbf{i}_P+\mathbf{i}_{PT}, \tag{15}$$

where

$$\mathbf{i}_P=\mathbf{i}_1^P+\mathbf{i}_2^P=\sigma_{\mathscr{H}}^{P1}\left\{\left[\left\langle\frac{\mathbf{s}}{\hbar}\right\rangle\langle\vec{\mathscr{H}}\rangle\right]+\alpha_0\left[\langle\vec{\mathscr{H}}\rangle\left[\left\langle\frac{\mathbf{s}}{\hbar}\right\rangle\langle\vec{\mathscr{H}}\rangle\right]\right]\right\} +\sigma_{\mathscr{H}}^{P2}\left\{\left[\left\langle\frac{\mathbf{I}}{\hbar}\right\rangle\langle\vec{\mathscr{H}}\rangle\right]+\alpha_0\left[\langle\vec{\mathscr{H}}\rangle\left[\left\langle\frac{\mathbf{I}}{\hbar}\right\rangle\langle\vec{\mathscr{H}}\rangle\right]\right]\right\}, \tag{16}$$

$$\mathbf{i}_{PT}=\mathbf{i}_1^{PT}+\mathbf{i}_2^{PT}=\sigma_{\mathscr{H}}^{PT1}\left\{\left[\left\langle\frac{\mathbf{I}}{\hbar}\right\rangle\left[\langle\vec{\mathscr{H}}\rangle\left\langle\frac{\mathbf{s}}{\hbar}\right\rangle\right]\right] +\alpha_0\left[\langle\vec{\mathscr{H}}\rangle\left[\left\langle\frac{\mathbf{I}}{\hbar}\right\rangle\left[\langle\vec{\mathscr{H}}\rangle\left\langle\frac{\mathbf{s}}{\hbar}\right\rangle\right]\right]\right]\right\}+\sigma_{\mathscr{H}}^{PT2}\left\{\left[\left\langle\frac{\mathbf{s}}{\hbar}\right\rangle \times\left[\left\langle\frac{\mathbf{I}}{\hbar}\right\rangle\langle\vec{\mathscr{H}}\rangle\right]\right]+\alpha_0\left[\langle\vec{\mathscr{H}}\rangle\left[\left\langle\frac{\mathbf{s}}{\hbar}\right\rangle\left[\left\langle\frac{\mathbf{I}}{\hbar}\right\rangle\langle\vec{\mathscr{H}}\rangle\right]\right]\right]\right\}. \tag{17}$$

Here

$$\sigma_{\mathscr{H}}^{P1}=(\hbar\mu_s/e)Q_1^P\sigma_e, \tag{18}$$

$$\sigma_{\mathscr{H}}^{P2}=(\hbar\mu_N g/e)Q_2^P\sigma_e, \tag{19}$$

$$\sigma_{\mathscr{H}}^{PT1}=(\hbar^2\mu_s/e)Q_3^{PT}\sigma_e, \tag{20}$$

$$\sigma_{\mathscr{H}}^{PT2}=(\hbar^2\mu_N g/e)Q_3^{PT}\sigma_e, \tag{21}$$

σ_e is the usual electric conductivity $\alpha_0=e\tau/mc$, τ is the time of collisional relaxation for the current. Putting $\langle\mathscr{H}\rangle\approx 10^3$ Oe and $\tau\approx 10^{-14}$ s (a value typical of metals), we get $\alpha_0\langle\mathscr{H}\rangle\ll 1$. The last condition was used in the derivation of (16) and (17). The first terms in each set of curly brackets in (16) and (17) determine the magnitudes and directions of the P- and PT-odd currents, while the second are small corrections and describe the Hall effect. The current $\mathbf{i}_2^{PT}$ is determined by the same constant $\tilde{Q}_3$ as the current $\mathbf{i}_1^{PT}$, and smaller than the latter by a factor $\mu_s/g\mu_N=10^3$. We shall therefore disregard it hereafter. The first term of (16) coincides with the one obtained earlier in Ref. 1.

5. We discuss now possible experiments. It can be seen from (16) and (17) that current can flow only so long as the direction of $\langle\mathbf{s}\rangle$ or $\langle\mathbf{I}\rangle$ does not coincide with the direction of the field $\langle\vec{\mathscr{H}}\rangle$, i.e., so long as there is no magnetic relaxation. In paramagnets, the direction of the vector $\langle\vec{\mathscr{H}}\rangle$ coincides then with the direction of the external field $\vec{\mathscr{H}}_{ext}$ while in ferromagnets it is necessary to take into account the intrinsic magnetic field and to consider $\langle\mathscr{H}\rangle=\langle\vec{\mathscr{H}}_{int}\rangle+\vec{\mathscr{H}}_{ext}$. The electron-spin magnetic-relaxation time τ_M^s in electricity conductors is determined by the interaction with the conduction electron and is of the order of $\tau_M^s\approx 10^{-9}$ s (Ref. 5). Thus, $\tau_M^s\gg\tau$ and for time interval of order τ_M^s the current can be regarded as a stationary process. The problem reduces hence to the possibility of experimentally observing a weak current within short time intervals. The relaxation time of nuclear spins in metals is substantially longer: $\tau_M^I\approx 10^{-3}$ s. We note also that the relaxation time increases in inverse proportion to the first degree of the temperature T.

Let us estimate the possible value of the current i_p. According to Ref. 1, for heavy metals (e.g., Pt) we have $\sigma_{\mathscr{H}}^{Pt}\approx 10^{-16}\,\sigma_e$. The effective field strength of the extraneous forces

$$\mathbf{E}_{ext}^{eff}=(\sigma_{\mathscr{H}}^{P1}/\sigma_e)\left[\left\langle\frac{\mathbf{s}}{\hbar}\right\rangle\mathbf{H}_{ext}\right] \tag{22}$$

turns out at $\mathscr{H}_{ext}\approx 10^3$ Oe to be $E_{ext}^{eff}\approx 10^{-11}$ V/cm. For a conductor 1 cm long and 0.1 cm^2 in cross section, having a typical metallic conductivity $\sigma_e\approx 10^5\ \Omega^{-1}\cdot\text{cm}^{-1}$ at room temperatures, we obtain a current on the order of $J\approx 10^{-7}$ A. Recognizing that, in order of magnitude [see (18) and (19)]

$$\sigma_{\mathscr{H}}^{P2}/\sigma_{\mathscr{H}}^{P1}\approx Z^{-1}\mu_N g/\mu_s, \tag{23}$$

which amounts to 10^{-5} for heavy metals, and bearing also in mind the relation $\sigma_e\sim T^4$, we obtain for the sample considered above the experimental results at $\mathscr{H}_{ext}\approx 10^3$ Oe, as indicated in the table. The experiments impose stringent requirements on the stability of the field $\vec{\mathscr{H}}$, since the effect of interest to us can be imitated by electromagnetic induction. According to estimates in Ref. 1, these restrictions on the current i_1^P at $\mathscr{H}\approx 10^3$ Oe are

$$d\mathscr{H}/dt<10^{-4}\ \text{Oe/s}. \tag{24}$$

For the current i_2^P we have correspondingly

$$d\mathscr{H}/dt<10^{-9}\ \text{Oe/s}. \tag{25}$$

One must add to these restrictions also the restrictions, not discussed in Ref. 1, on the rate of rise time of the pulse of the field $\mathscr{H}_{ext}$: the duration of the passage of the front should be shorter by at least an order than the time τ_M. The rise time of the pulse must therefore satisfy at $\mathscr{H}\approx 10^3$ Oe the condition

$$d\mathscr{H}/dt>10^4/\tau_M\ [\text{Oe}], \tag{26}$$

TABLE I.

T, K	i_1^P, A	τ_M^s, s	i_2^P, A	τ_M^I, s
300 10	10^{-7} 10^{-3}	10^{-9} 10^{-8}	10^{-12} 10^{-8}	10^{-3} 10^{-1}

whereas the conditions (24) and (25) must be satisfied on the plateau.

We must also exclude the influence of the additional short-circuit currents that attenuate with time like $I_e = I_0 \exp\{-Rt/L\}$, where R and L are the resistance and inductance of the circuit with sample, while I_0 is determined by the value of $d\mathscr{H}/dt$ from (26). An estimate shows that to satisfy the condition $I_e \ll i^P_{1,2}$ we must stipulate $R\tau_M/L \approx 10^2$. This leads to a condition on the inductance L, which must be made small enough. It must be remembered, finally, that the vectors $\langle \mathbf{s} \rangle$ and $\langle \mathbf{I} \rangle$ precess in the course of damping at a frequency $\omega_s = e\mathscr{H}/mc$ in the case of the current $\mathbf{i}^P_1$ are with a frequency $\omega_I = (\mu_N g/\mu_s)\omega_s$ in the case of the current $\mathbf{i}^P_2$. For $\mathscr{H} \approx 10^3$ Oe we get $\omega_s \approx 10^{10}$ Hz and $\omega_I \approx 10^7$ Hz, i.e., $\omega_s \approx 2\pi/\tau^s_M$, $\omega_I \gg 2\pi/\tau^I_M$. Thus, periodic damping can occur in the case of the current $\mathbf{i}^P_1$, while for the current $\mathbf{i}^P_2$ the process has high frequency.

Note that experiments with pulses of various durations and with different carrier frequencies make it possible in principle to determine currents $\mathbf{i}^P_1$ and $\mathbf{i}^P_2$ connected with different constants of the effective weak Hamiltonian.

We conclude by considering the possibility of observing the current $\mathbf{i}^{PT}_{1,2}$. One can expect from an analysis of the existing models that these currents are weaker than $\mathbf{i}^P_{1,2}$ by 10^2–10^9 times.[6] It is therefore important in principle that they not be masked by the currents $\mathbf{i}^P_{1,2}$. From (16) and (17), for example, it follows that at $\langle \mathbf{s} \rangle \perp \mathscr{H}_{ext}$ the Hall current in (16) is directed along $\langle \mathbf{s} \rangle$. Therefore, assuming that $\langle \mathbf{I} \rangle \| \langle \mathbf{s} \rangle$ in the magnetic field, we conclude that the current $\mathbf{i}^{PT}_1$ is directed in this case along the vector $\vec{\mathscr{H}}_{ext}$. It is thus possible in principle to separate the P- and PT-odd effects.

The authors thank V. G. Gorshkov, D. A. Kirzhnits, M. G. Kozlov, and I. B. Khriplovich for numerous discussions and suggestions.

[1)]No direct current is actually produced in a constant magnetic field, but parity-nonconservation effects can exist in the form of pulsed currents.

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Translated by J. G. Adashko