Homework 5

Phys 556: Solid State II
Assigned: April 5, Due: April 16

1 Email me a topic for your final paper/presentation.

The list of suggested papers and guidelines for the presentation and paper can be found on my website: `https://you.stonybrook.edu/jcano/phys556-2019-final/`

Papers are distributed on a first-come, first-serve basis. You are welcome to suggest a paper. It should be a somewhat classic or foundational paper. It is subject to my approval.

2 Generalized Landau theory: $u < 0$ (Adapted from Exercise 11.3 in Ref. [1])

Consider the more general class of Landau theory where the interaction u can be negative:

$$f[\psi] = \frac{r}{2}\psi^2 + u\psi^4 + u_6\psi^6 - h\psi \tag{1}$$

(a) Show that for $h = 0, u < 0, r > 0$, the free energy contains three local minima, one at $\psi = 0$ and two others at $\psi = \pm\psi_0$, where

$$\psi_0^2 = -\frac{u}{3u_6} \pm \sqrt{\left(\frac{u}{3u_6}\right)^2 - \frac{r}{6u_6}}. \tag{2}$$

(b) Show that for

$$r < r_c \equiv \frac{u^2}{2u_6}, \tag{3}$$

the solution at $\psi = 0$ becomes metastable. Solve for r_c by setting $f[\psi_0] = f[0] = 0$. Show that, unlike the $u_6 = 0$ case discussed in class, ψ jumps discontinuously across the phase transition.

(c) Assume $r = a(T - T_c)$. Sketch the (T, u) phase diagram for $h = 0$ and fixed $u_6 > 0$.

(d) Consider $r = 0, h \neq 0$. Solve for ψ in the limiting cases $|h| \gg |u|$ and $|h| \ll |u|$.

(e) Argue that there are two critical lines in the $r = 0$ plane which cross at $h = u = 0$.

3 Coherent states (Adapted from Ref. [2])

Let $b^\dagger$ be a boson creation operator. Given a complex number, α, define the coherent state $|\alpha\rangle$ by:

$$|\alpha\rangle = e^{-|\alpha|^2/2} e^{\alpha b^\dagger}|0\rangle = e^{-|\alpha|^2/2} \sum_{N=0}^{\infty} \frac{\alpha^N}{\sqrt{N!}}|N\rangle, \tag{4}$$

where

$$|N\rangle \equiv \frac{1}{\sqrt{N!}}(b^\dagger)^N|0\rangle. \tag{5}$$

(a) Prove that $|\alpha\rangle$ is normalized.

(b) Prove that $|\alpha\rangle$ is an eigenstate of b with eigenvalue α.

(c) Prove that the set $\{|\alpha\rangle\}$ spans the space $\{|N\rangle\}$. (In other words, show that each $|N\rangle$, $N \geq 0$, can be written as a superposition of $\{|\alpha\rangle\}$.)

(d) Compute $\overline{N} = \langle\alpha|\hat{n}|\alpha\rangle$ and $\overline{N^2} = \langle\alpha|\hat{n}^2|\alpha\rangle$ and show that $\Delta N = \sqrt{\overline{N^2} - (\overline{N})^2} \ll \overline{N}$ when $\overline{N} \gg 1$. Therefore, in a coherent states with $\overline{N} \gg 1$, b and $b^\dagger$ behave approximately like classical variables, that is, $\langle[b, b^\dagger]\rangle \ll \overline{N}$.

References

[1] "Introduction to Many-Body Physics" by Piers Coleman.

[2] "Modern Condensed Matter Physics" by Steven Girvin and Kun Yang.