Math Club's 3 Problems Week 14*

September 4, 2019

1. Let a, b, and c be positive real numbers and $abc = 1$. Prove that

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \geq \frac{3}{2}.$$

2. For any two real numbers a and b consider the sequence

$$p_n = [2\{an + b\}], \quad n = 0, 1, 2, \ldots$$

(where $\{x\}$ and $[x]$ denote the fractional and integer parts of a number x, respectively). Any set of successive terms of this sequence will be called a word. Is it true that any ordered set of k zeros and ones is a word in the sequence p_n for a certain pair of numbers a and b a) $k = 4$,(b) $k = 5$?

3. The osculating circle of a C^2 curve at a point x is acquired by taking three points on the curve, finding the (unique) circle that passes through those three points, and finding the limit circle as the three points approach x. Show that

   (a) the osculating circle has the same tangent and the same curvature at x as the curve.

   (b) the osculating circles of an arc with monotonic positive curvature are pairwise disjoint and nested.

*Email your solutions to us at sbumathclub@gmail.com (and/or) present it during our GBM on the next Wed. at 7pm at Math Tower p-131.