Math Club Guest Talk: Counting prime paths in graphs

Dmitry Zakharov

Wednesday, April 22, 2020 7PM Zoom (online)

The Riemann zeta function can be thought of as enumerating the prime numbers, and is the subject of possibly the single most important unsolved problem in mathematics. The Ihara zeta function is a graph-theoretic analogue of the Riemann zeta function. Instead of counting prime numbers, it enumerates prime paths, which are defined as closed paths satisfying some restrictions. It turns out that, unlike the Riemann zeta, the Ihara zeta function has a simple expression as the reciprocal of a polynomial. I will explain how this formula is derive, and how certain properties of the graph can be determined from its zeta function. Finally, I will explain how the zeta function behaves under group actions on the graph.

The talk will require no mathematical background other than linear algebra and some elementary group theory.