An Elementary Proof that $CAT(\kappa)$ Surfaces have Bounded Integral Curvature

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The field of Alexandrov Geometry studies the geometric properties of special metric spaces where properties such as angle are well-defined. Our project studied $CAT(\kappa)$ surfaces. These are spaces in which small geodesic triangles are thinner than triangles drawn with matching edge lengths in a surface of constant curvature (i.e. a sphere, euclidean plane, or hyperbolic disk). This inequality gives a curvature bound on the surface. A separate way to give a curvature bound on a surface is to find a uniform upper bound on the angle excess of any finite collection of non-overlapping triangles contained in any given neighborhood of a surface. This condition is known as bounded integral curvature. In our project we proved that a $CAT(\kappa)$ surface is also a surface of bounded integral curvature using a technique called vertex-edge triangulation. This comes from the observation that angle excess of a triangulation monotonically increases and that model area of a triangulation monotonically decreases under subdivision. By a Gauss–Bonnet type argument, we derive a uniform upper bound on angle excess of a vertex edge triangulation. To our knowledge, this is a new result, although a proof is sketched in Machigashira–Ohtsuka (2001). In this talk we will discuss our method of proof, and will also share some highlights of our REU experience.