Making Markov Chains With Metropolis

Daniel Waxman
P131 Math Tower
Tuesday 7:00 PM September 27, 2022

A common problem in statistical physics and Bayesian inference is to numerically estimate "expectations," or integrals with respect to a probability distribution. An intuitive family of approaches to this is that of "Monte Carlo methods," which estimate these integrals with random samples. Monte Carlo methods in their modern form date back to the Second World War and the Manhattan Project, with "Markov Chain Monte Carlo" (MCMC) being born shortly after with the work of Metropolis et al. (1953). MCMC methods rely on a surprising and clever trick, and the bulk of this talk will be exploring a mathematical justification of MCMC, namely the so-called "Metropolis-Hastings" method.

To do this, we'll introduce stochastic processes. These are collections of random variables, usually indexed with time, with a specified dependence on each other. One of the simplest dependencies is the "Markov property," where each variable depends only on the state at the previous time instant. If the process is indexed discretely, we may call it a "Markov chain." We'll introduce Markov chains, their dynamics, what their "stationary distributions" are, and how this all relates to Metropolis-Hastings.