A first look at percolation theory

Kurt Butler Math Tower, P-131 7:00 pm on Thursday, February 23rd, 2023

Imagine an infinite forest that is organized like a giant chess board. For each square of the board, there is a probability p that a tree occupies that square. If a bad actor ignites one of the trees, we want to understand how big the forest fire will become, assuming that the fire can only spread to trees on adjacent squares. When p > 0.5, there almost-surely exists an infinitely large cluster of trees, and so igniting one of these trees can produce an infinitely large fire. If p < 0.5, an infinite forest fire is almost-surely impossible. What is it about this model that makes the p > 0.5 and p < 0.5 cases so different?

This talk will be a friendly introduction to the percolation model. This model is well-studied in probability theory and leads to lots of interesting problems to think about, but their solutions often require clever mathematics and some fun pictures.