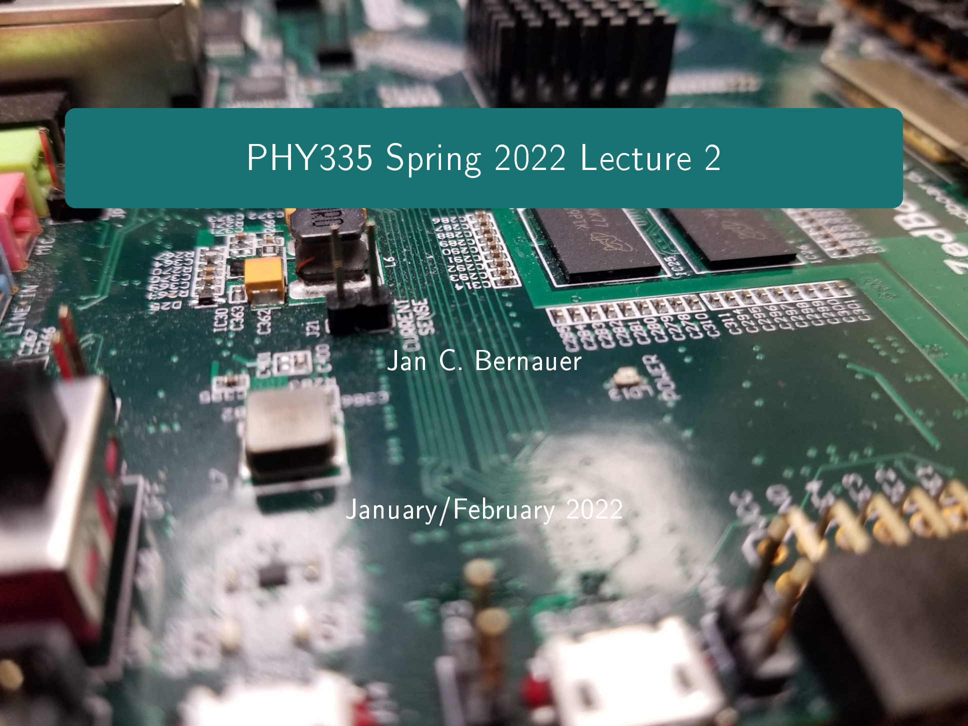

PHY335 Spring 2022 Lecture 2

Jan C. Bernauer

January/February 2022

Linear passive components

There are exactly three linear passive components.

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- Resistors

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- Capacitors

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- Resistors
- Capacitors
- Inductors

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- Two large area ”plates” with some insulator between

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$$Q = C \cdot V$$

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- Simple parallel plate:

$$C = \epsilon \frac{A}{d} = k\epsilon_0 \frac{A}{d}$$

- Unit of capacitance (Farad):

$$[C] = 1F = \frac{1C}{1V} = \frac{1As}{1V}$$

Capacitors and signals

$$Q = C \cdot V \longrightarrow I = C\frac{dV}{dt}$$

- Typical values are pF to μF

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- Typical values are pF to μF
- Example: A 10ms long current pulse of 1 mA into a $1\mu F$ will change the voltage by:

$$\Delta V = \frac{1mA \cdot 10ms}{1 \cdot 10^{-6}F} = 10V$$

Capacitors = energy storage

- Power:

$$P = VI = VC\frac{dV}{dt}$$

Capacitors = energy storage

- Power:

$$P = VI = VC\frac{dV}{dt}$$

- Stored energy:

$$E = \int_0^{V_{max}} VCdV = \frac{1}{2}CV^2$$

- The power into a resistor ends up as heat. The power into a capacitor is stored in the electrical field!

Capacitors in parallel

$$C_{total} V = Q_{total} = Q_1 + Q_2$$

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Capacitors in series

$$Idt = C_1 dV_1 = C_2 dV_2 = C_{total} dV_{total}$$

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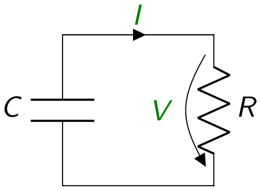
$$C_{total} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_{2\,1}}$$

RC Circuits: V and I vs time



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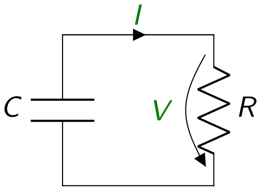
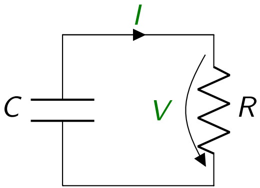


$$I = \frac{V}{R}$$

RC Circuits: V and I vs time



$$I = \frac{V}{R} = -C\frac{dV}{dt}$$

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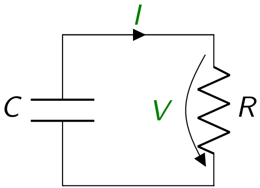

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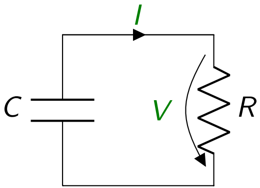

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Assume capacitor is charged to V_0 at $t = 0$:

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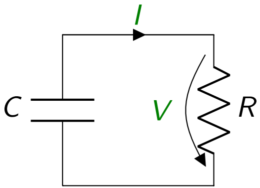

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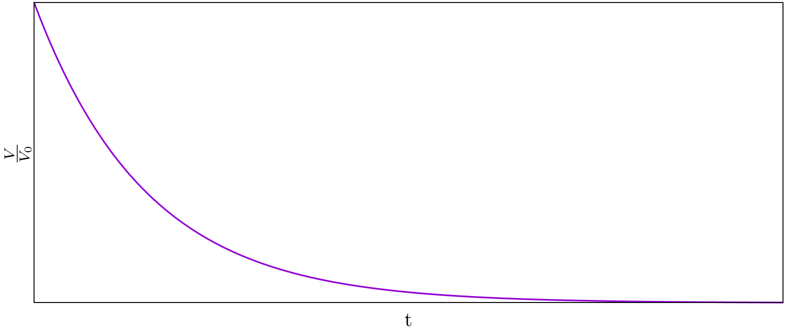
$$[RC] = \frac{V}{A} \cdot \frac{C}{V} = \frac{As}{A} = s$$

Meaning of RC



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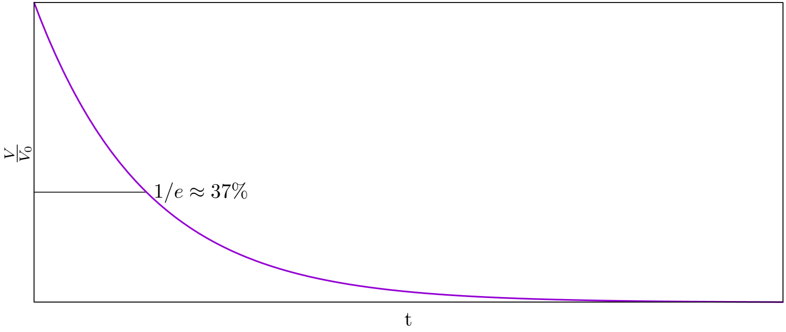

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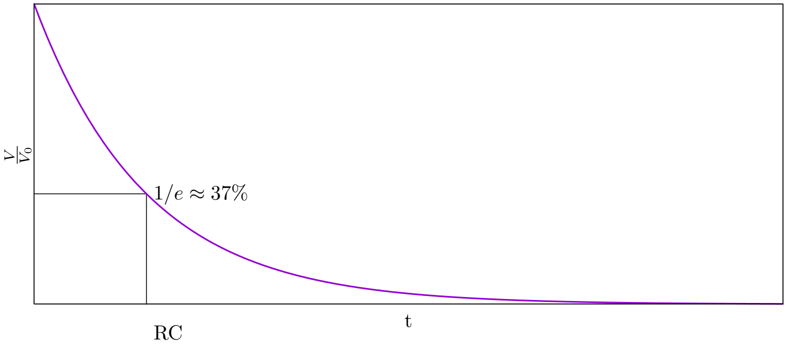

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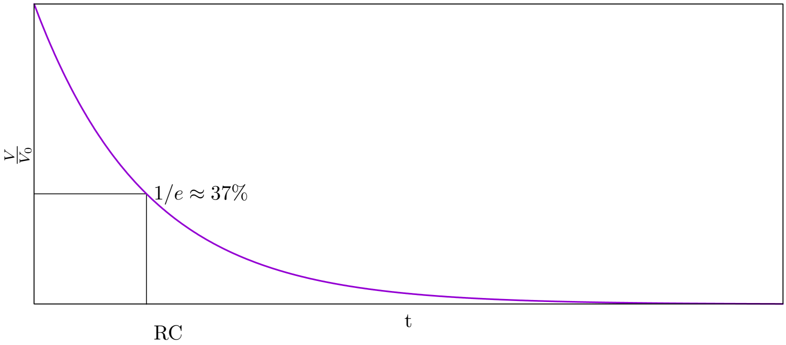

RC is the time in which the signal closes in to the $T = \infty$ value by 63%.

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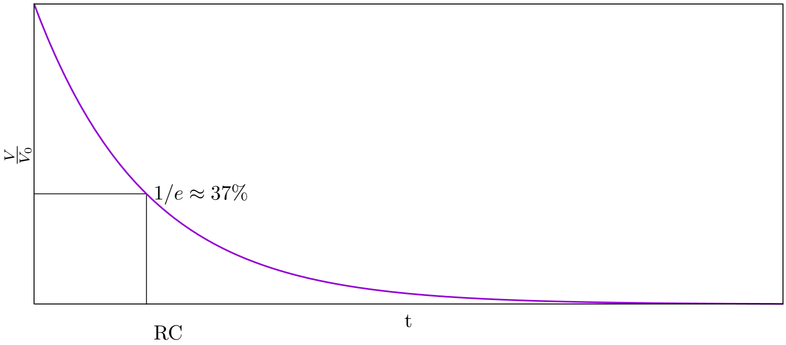
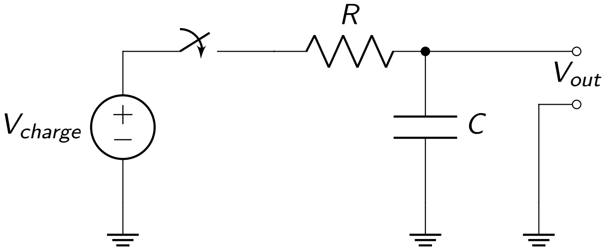


RC is the time in which the signal closes in to the $T = \infty$ value by 63%.

We say: RC is the time constant of the circuit!

Another example



The capacitor is not charged. We close the switch at $t = 0$.

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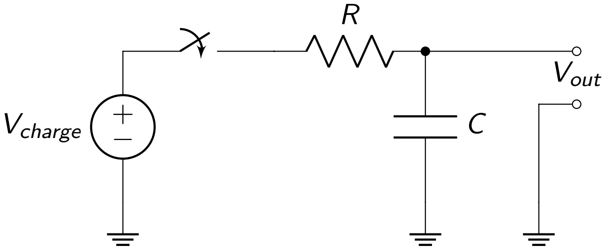

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$$I = C\frac{dV_{out}}{dt}$$

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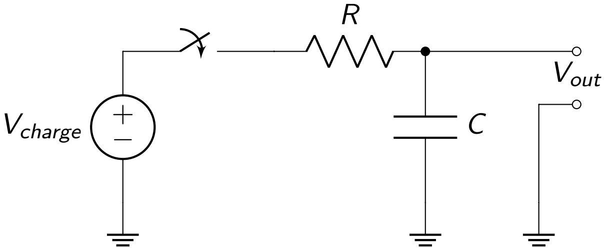

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$$I = C\frac{dV_{out}}{dt} = \frac{V_{charge} - V_{out}}{R}$$

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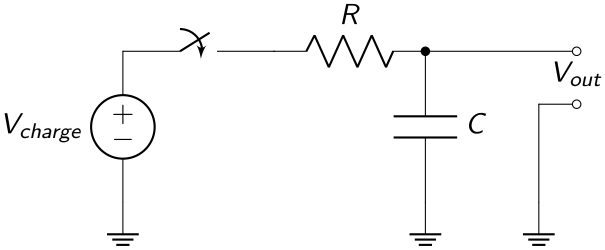
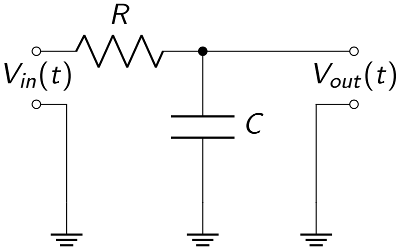


The capacitor is not charged. We close the switch at $t = 0$.

$$I = C\frac{dV_{out}}{dt} = \frac{V_{charge} - V_{out}}{R}$$

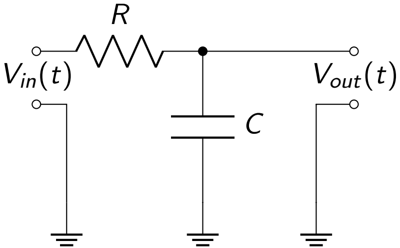
$$V_{out} = V_{charge}\left(1 - e^{-t/RC}\right)$$

Integrator



What happens if we replace V_{charge} with a time dependent $V_{in}(t)$?

Integrator

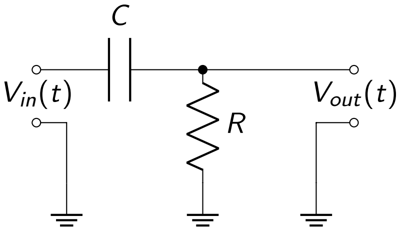


What happens if we replace V_{charge} with a time dependent $V_{in}(t)$?

$$V_{out}(t) = \frac{1}{RC} \int_{\infty}^{t} V_{in}(\tau) e^{-\frac{t-\tau}{RC}} d\tau$$

For large RC: Integration!

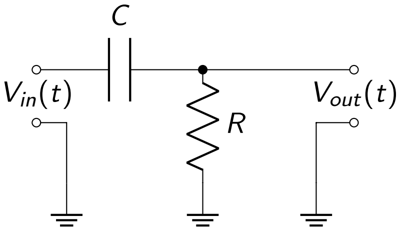
Differentiator



Let's flip R and C.

$$I = \frac{V_{out}(t)}{R}$$

Differentiator



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$$I = \frac{V_{out}(t)}{R} = C\frac{d}{dt}(V_{in}(t) - V_{out}(t))$$

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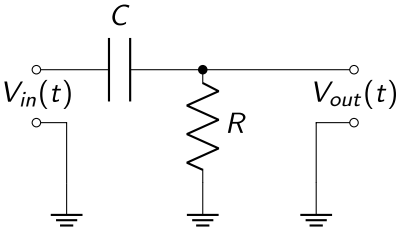

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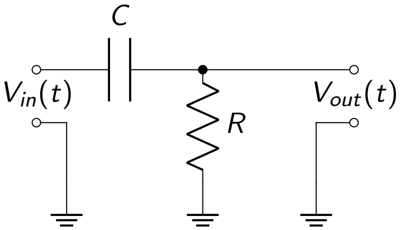

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$$V_{out}(t) = RC\frac{d}{dt}(V_{in}(t) - V_{out}(t))$$

For small RC:

$$V_{out}(t) \approx RC\frac{d}{dt}V_{in}(t)$$

Inductors

The voltage over a coil depends on the rate of change of the current:

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A coil stores energy in the magnetic field.

$$E = \frac{1}{2}LI^2$$

Unit of inductance (Henry):

$$[L] = \frac{Vs}{A} = H$$

Linear devices

- R, L, and C are linear devices.

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- R, L, and C are linear devices.
- One consequence: For given waveform, output amplitude and input amplitude have fixed ratio.
- But not necessarily same shape!
- Sine waves stay sine waves, but with a phase:
  - R produces (generally) no phase shift
  - L and C do, because
    $\frac{d}{dt}\sin(\omega t) = \omega\cos(\omega t) = \omega\sin(2\pi f t + 90^\circ)$

Let's go complex

Instead of keeping track of phases, we can express a signal as a complex value, with complex amplitudes, and (later) only look at the real part.

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Example:

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We can encode Amplitude + phase into a complex amplitude $\boldsymbol{A}$.

Real amplitude and phase from complex amplitude:

$$\Re[\boldsymbol{A}e^{j\omega t}] = \Re[\boldsymbol{A}]\cos\omega t - \Im[\boldsymbol{A}]\sin\omega t$$

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$$\Re[\boldsymbol{A}e^{j\omega t}] = \Re[\boldsymbol{A}]\cos\omega t - \Im[\boldsymbol{A}]\sin\omega t$$

$$= \sqrt{\Re[\boldsymbol{A}]^2 + \Im[\boldsymbol{A}]^2}\cos(\omega t + \phi)$$

$$\phi = atan2(\Im[\boldsymbol{A}], \Re[\boldsymbol{A}])$$

Some tricks with complex numbers

$$\frac{1}{a + jb} = \frac{a - jb}{a^2 + b^2}$$

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$$\frac{1}{a+jb} = \frac{a-jb}{a^2+b^2}$$

$$|\frac{1}{a+jb}| = \sqrt{\frac{a-jb}{a^2+b^2} \cdot \frac{a+jb}{a^2+b^2}} = \sqrt{\frac{a^2+b^2}{(a^2+b^2)^2}} = \frac{1}{\sqrt{a^2+b^2}}$$

Impedance: Complex resistance

We can now introduce the complex resistance Z, called Impedance

$$\boldsymbol{Z} = R + jX$$

$$\text{Impedance} = \text{Resistance} + \text{Reactance}$$

Recovering Ohm's law: Capacitor

Let's assume we have a (co)sine wave with an angular frequency ω and (real) amplitude A:

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So we find an impedance of

$$\boldsymbol{Z_C} = \frac{\boldsymbol{V}(t)}{\boldsymbol{I}(t)} = \frac{1}{j\omega C}$$

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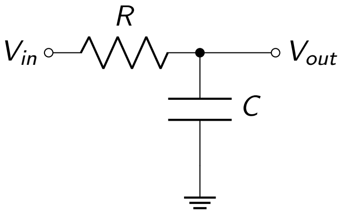
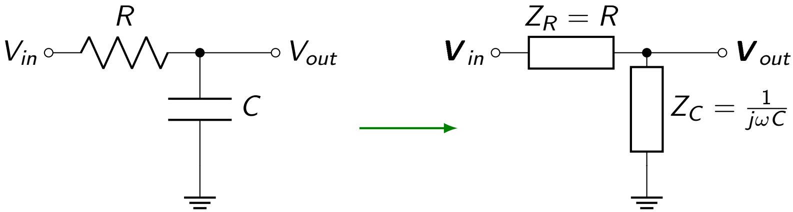
So we find an impedance of

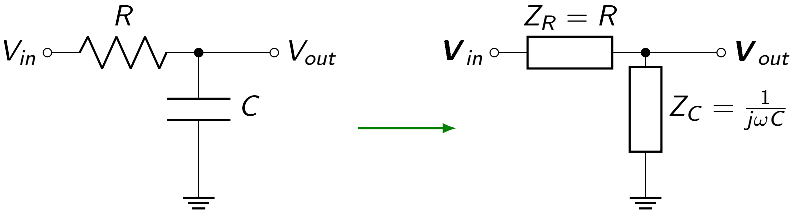
$$\boldsymbol{Z_L} = \frac{\boldsymbol{V}(t)}{\boldsymbol{I}(t)} = j\omega L$$

Low-pass filter




R
V_{in}
V_{out}
C
$Z_R = R$
$\boldsymbol{V}_{in}$
$\boldsymbol{V}_{out}$
$Z_C = \frac{1}{j\omega C}$

Low-pass filter



Voltage divider!

$$\boldsymbol{G}(\omega) = \frac{\boldsymbol{V}_{out}}{\boldsymbol{V}_{in}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega/\omega_0}$$

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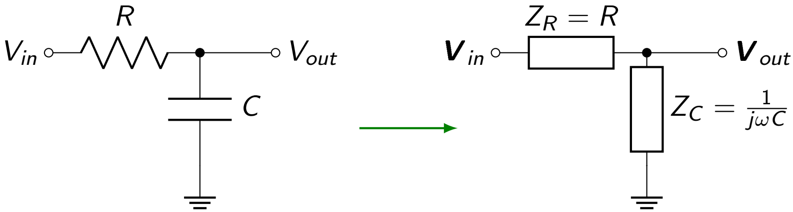

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Amplitude ratio:

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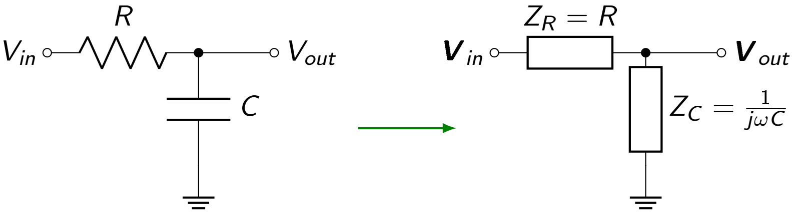

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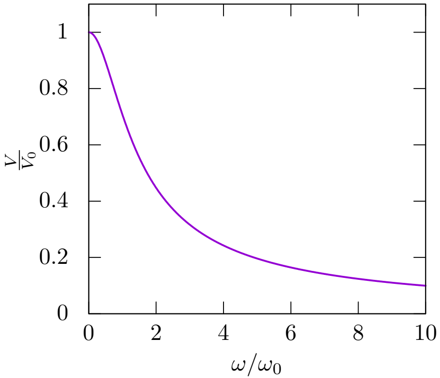
Phase shift:

$$\phi = -\tan^{-1} \omega RC = -\tan^{-1} \frac{\omega}{\omega_0}$$

Frequency response of a low pass filter (Bode plots)



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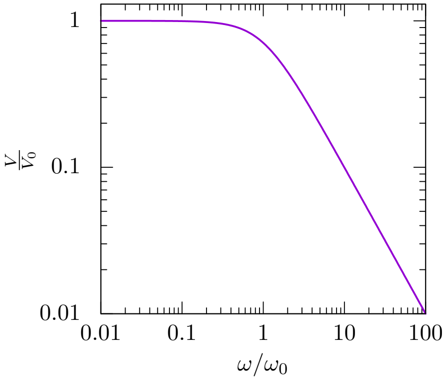

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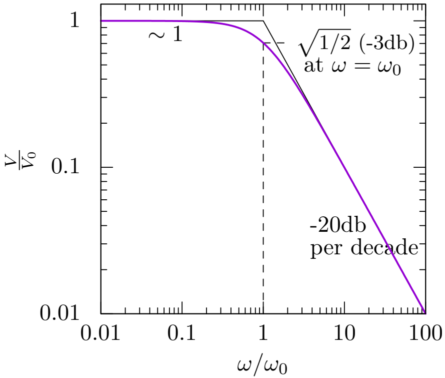

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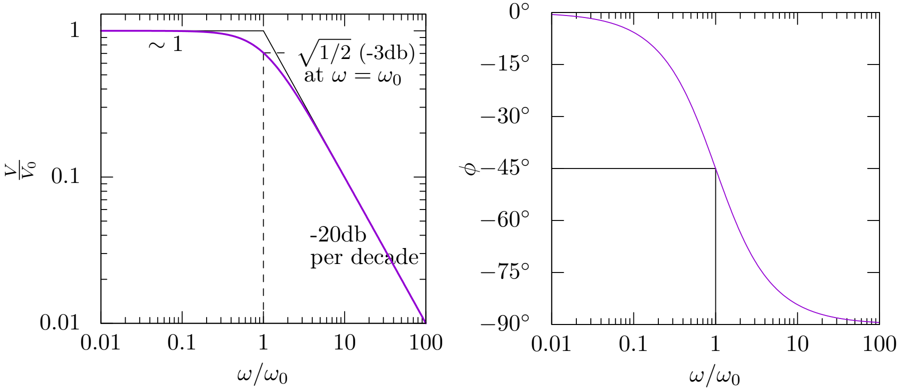

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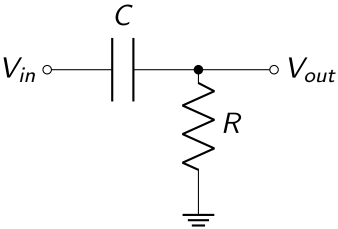

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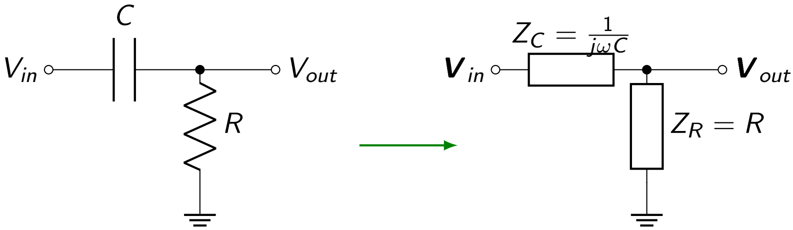

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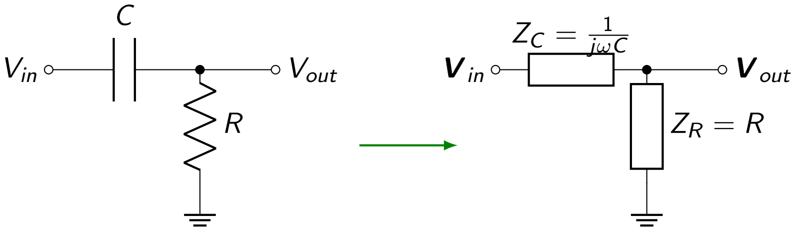

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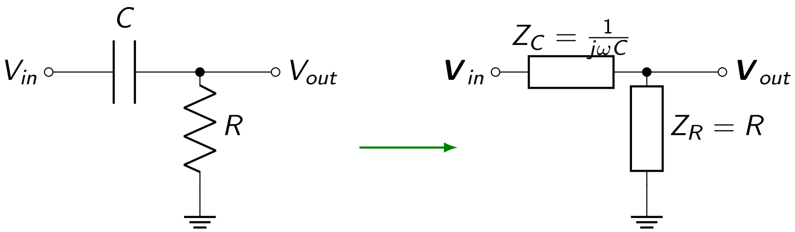

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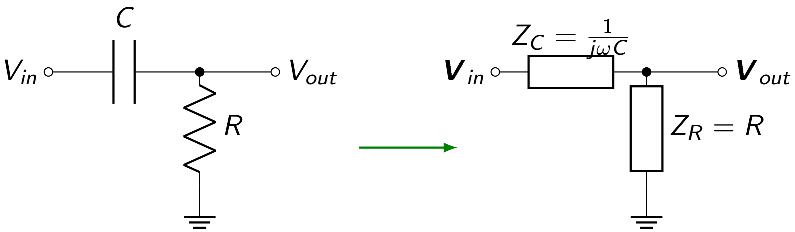
Amplitude:

$$\frac{V_{out}}{V_{in}} = \frac{\omega RC}{\sqrt{1 + \omega^2 R^2 C^2}} = \frac{\omega/\omega_0}{\sqrt{1 + (\omega/\omega_0)^2}}$$



Voltage divider!

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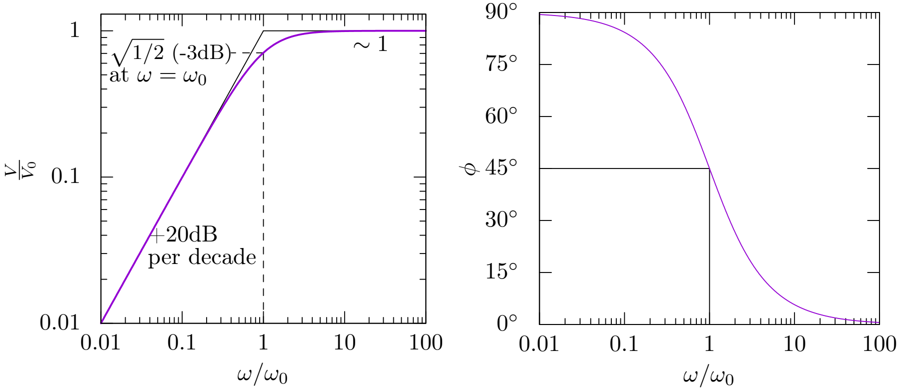
Phase shift:

$$\phi = \tan^{-1} 1/\omega RC = \tan^{-1} \frac{\omega_0}{\omega}$$

Frequency response of a high pass filter (Bode plots)



Phasers

Phas~~e~~ors

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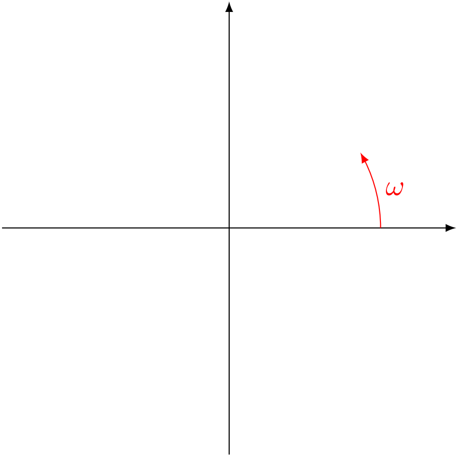
Phasors are a visual way to handle complex numbers.

- Draw vectors in complex plane
- Addition is vector addition
- Multiplication is
  - Multiplication of length
  - Addition of angle

Phasors



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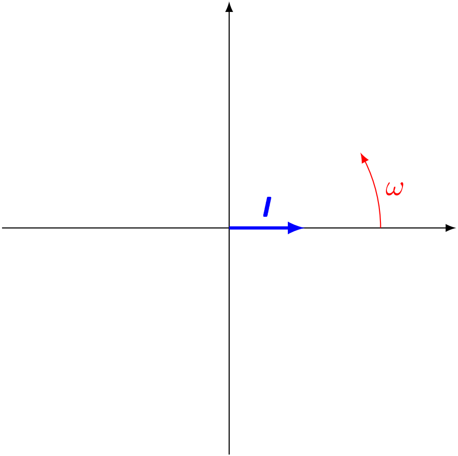

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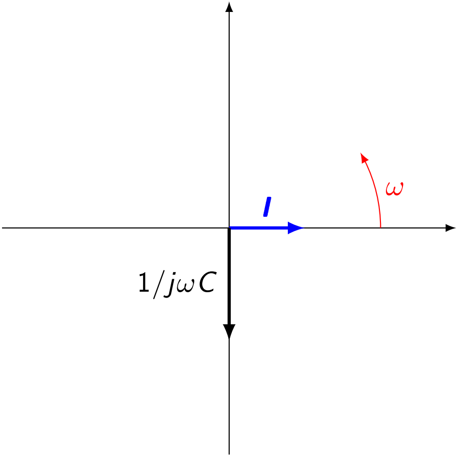

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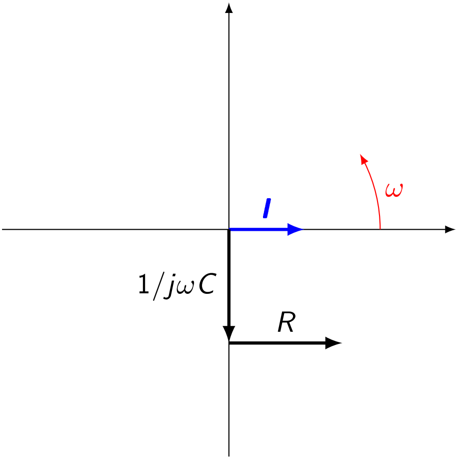

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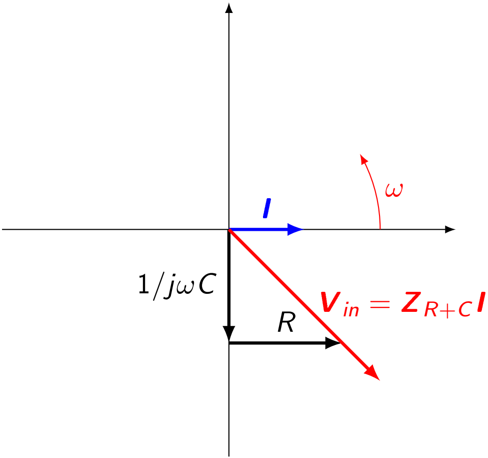
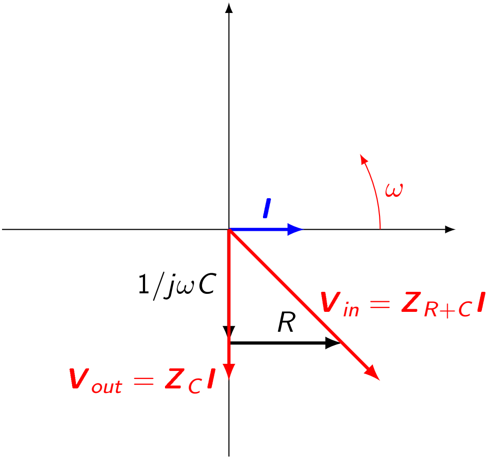
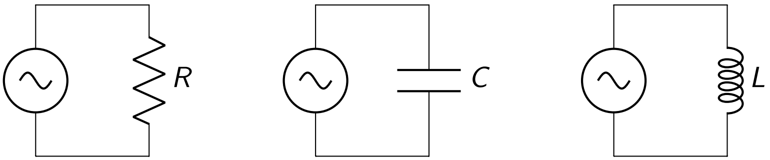


$\boldsymbol{I}$
ω
$1/j\omega C$
R
$\boldsymbol{V}_{in} = \boldsymbol{Z}_{R+C}\boldsymbol{I}$

Phasors

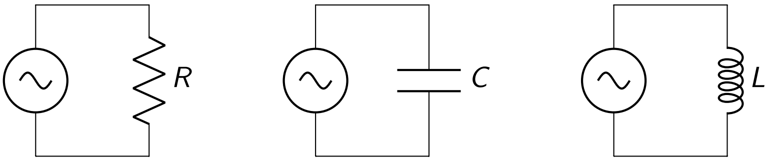


Power in L and C circuits



$$V_{in} \propto \sin \omega t$$

Power in L and C circuits



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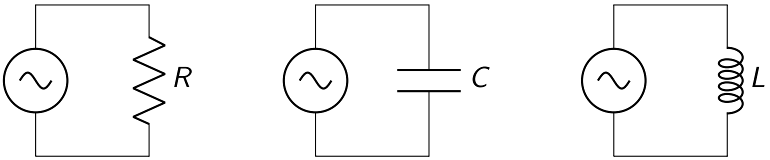

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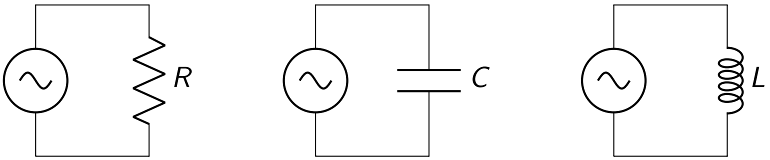


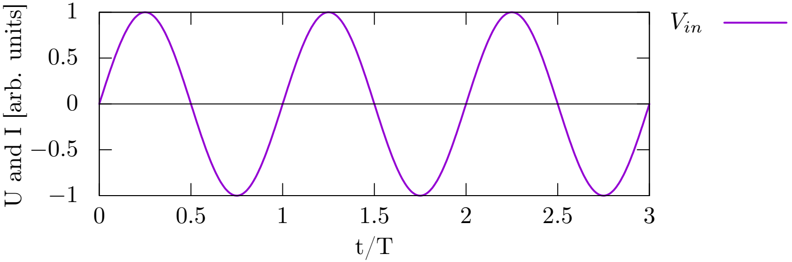
$$V_{in} \propto \sin \omega t$$

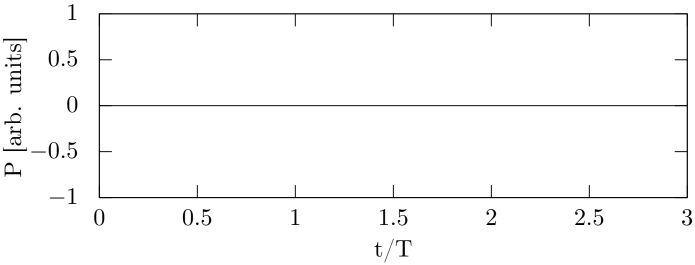
$$I_R \propto \sin \omega t$$

$$I_C \propto \frac{dV}{dt} \propto \cos \omega t$$

$$I_L \propto \int V dt \propto -\cos \omega t$$

Instantaneous power





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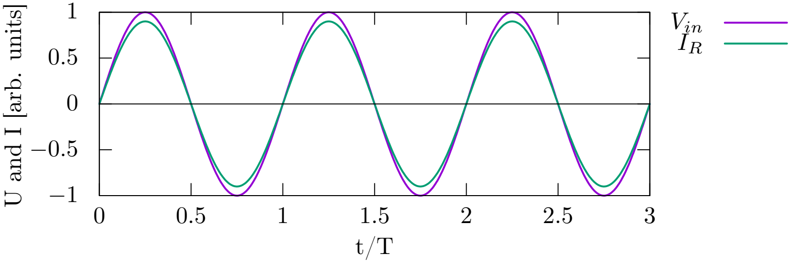

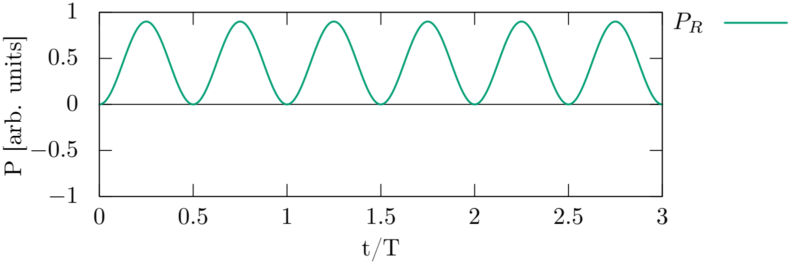

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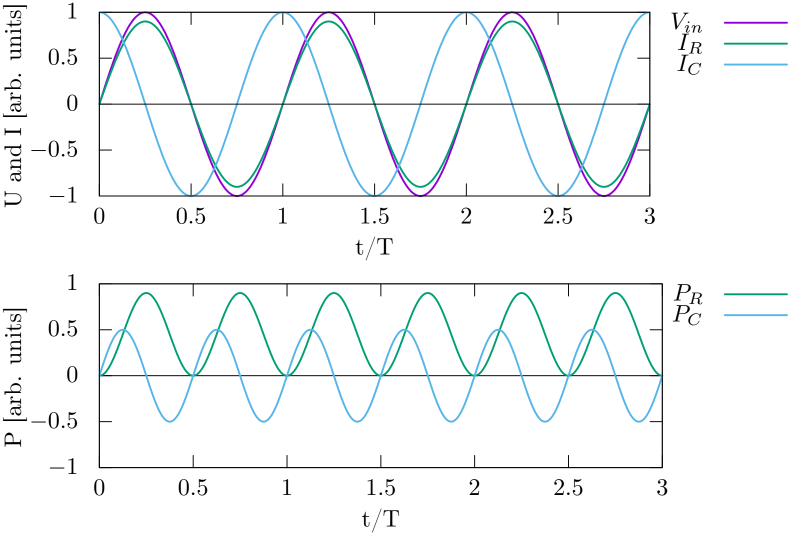

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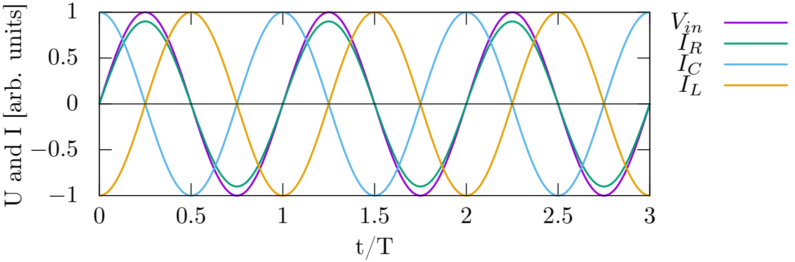

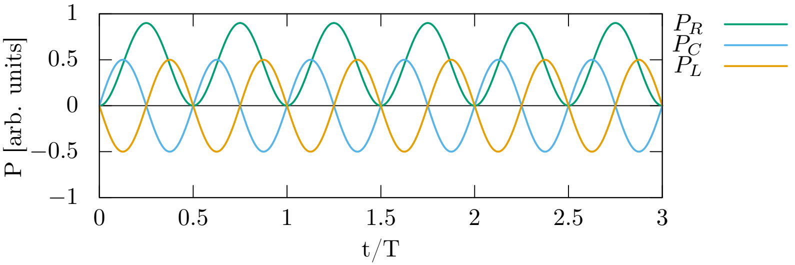

Average power

So while

$$\overline{P}_R = V_{R,rms} I_{R,rms}$$

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Average power

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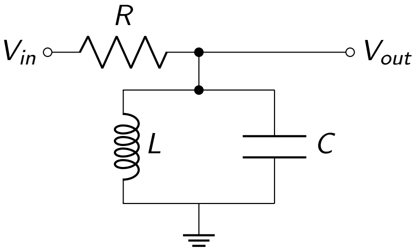
$$\overline{P}_L = \overline{P}_C = 0$$

The actual delivered power is called active power, measured in Watts. $P_A = V_{rms} I_{rms}$ is the "apparent power", often VA instead of Watts.

- Residential customers pay for active power
- Commerical customers often for apparent power.

LC circuits I: Tank circuit



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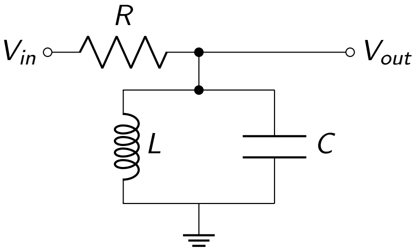

$$Z_{LC} = \frac{1}{\frac{1}{j\omega L} + j\omega C} = \frac{j}{(\frac{1}{\omega L} - \omega C)}$$

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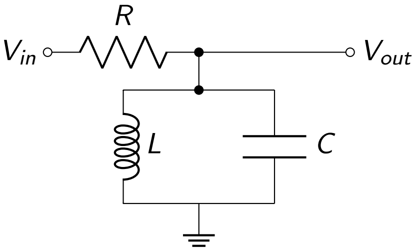

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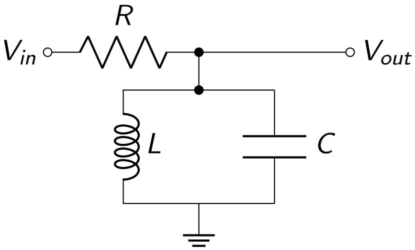

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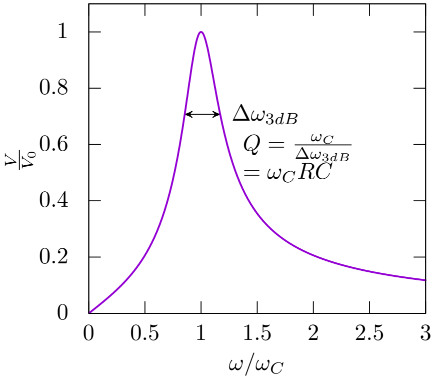
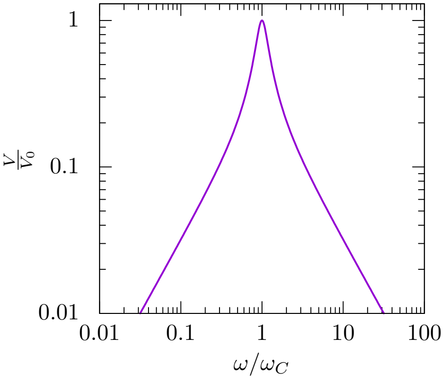
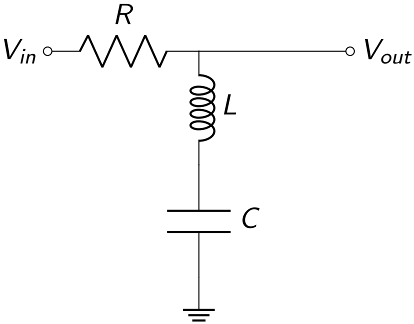
$$(\frac{1}{\omega_C L} - \omega_C C) = 0 \longrightarrow \omega_C = \sqrt{\frac{1}{LC}}$$

Frequency response of a tank circuit





LC circuits II: Notch filter



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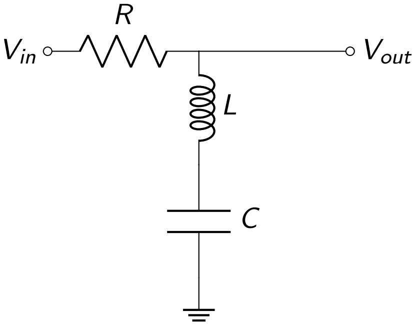

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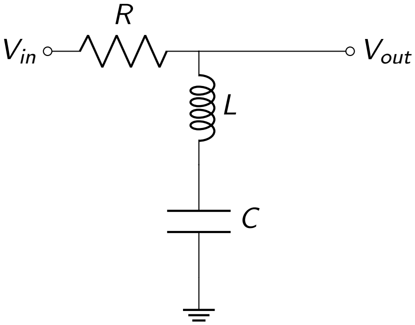

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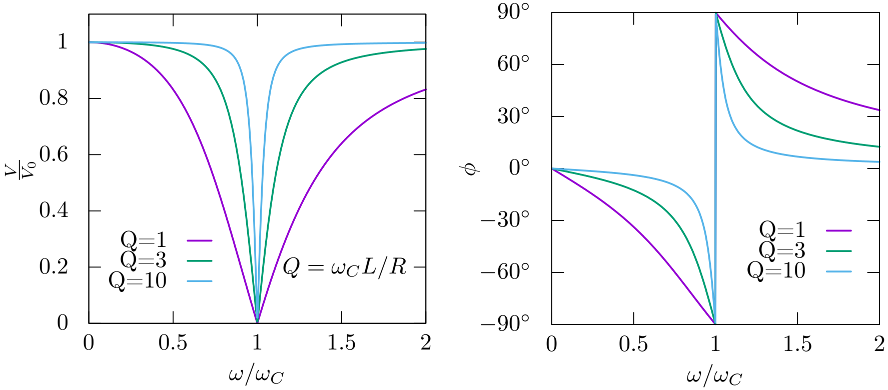

Transmission lines / cables

We often want to transfer signals from one location to another. For that we need cables. These are mostly coaxial cables.

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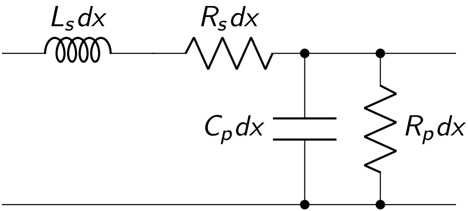
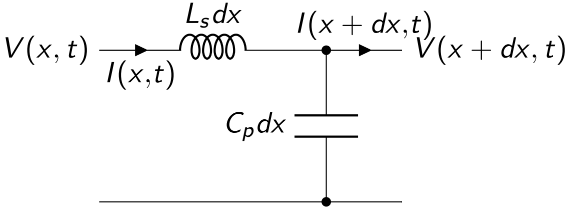
A coax cable has an inner conductor, which is surrounded by an isolator, surrounded by a shield.



Coax cable (simplified)

We can assume that $R_s$is small, and $R_p$is large.



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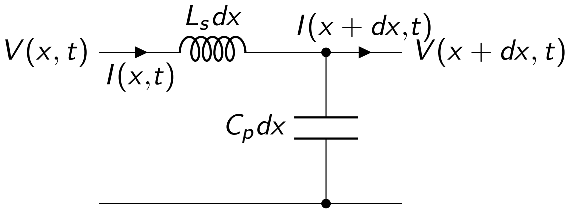

$$C_p dx \frac{dV(x + dx, t)}{dt} = I(x, t) - I(x + dx, t)$$

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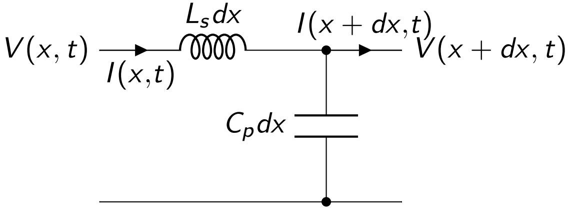

$$C_p dx \frac{dV(x+dx,t)}{dt} = I(x,t) - I(x+dx,t)$$

$$C_p dx \frac{d(V(x,t) + \frac{dV}{dx}(x,t)dx)}{dt} = I(x,t) - I(x+dx,t)$$

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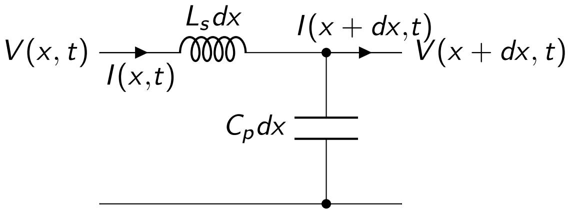

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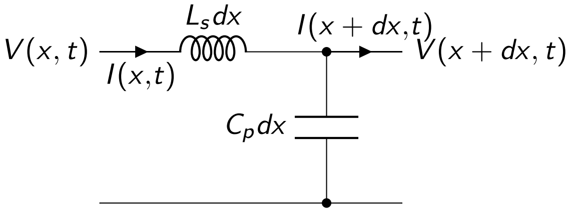
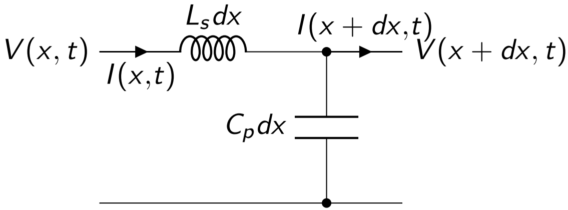
$$C_p \frac{dV(x,t)}{dt} = -\frac{dI(x,t)}{dx}$$

Coax cable II



$$C_p \frac{dV(x,t)}{dt} = -\frac{dI(x,t)}{dx}$$

$$L_s dx \frac{dI(x,t)}{dt} = V(x,t) - V(x+dx,t)$$



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Coax cable III

$$C_p \frac{dV(x,t)}{dt} = -\frac{dI(x,t)}{dx}$$

$$L_s \frac{dI(x,t)}{dt} = -\frac{dV(x,t)}{dx}$$

Differentiate the first with regard to dt, the second with regard to dx

Coax cable III

$$C_p \frac{dV(x,t)}{dt} = -\frac{dI(x,t)}{dx}$$

$$L_s \frac{dI(x,t)}{dt} = -\frac{dV(x,t)}{dx}$$

Differentiate the first with regard to dt, the second with regard to dx

$$C_p \frac{d^2V(x,t)}{dt^2} = -\frac{d^2I(x,t)}{dxdt}$$

$$L_s \frac{d^2I(x,t)}{dtdx} = -\frac{d^2V(x,t)}{dx^2}$$

Coax cable IV

$$C_p \frac{d^2 V(x,t)}{dt^2} = \frac{1}{L_s} \frac{d^2 V(x,t)}{dx^2}$$

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We can guess a solution: $V(x,t) = A\cos(\omega(t - \frac{1}{c}x))$

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$$-LC\omega^2 V(x,t) = -\frac{1}{c^2}\omega^2 V(x,t)$$

Coax cable IV

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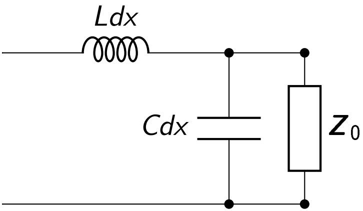
$$-LC\omega^2 V(x,t) = -\frac{1}{c^2}\omega^2 V(x,t)$$

$$c = \sqrt{\frac{1}{LC}}$$

The propagation speed in a coax cable is $c = \sqrt{1/LC} \approx \frac{20cm}{ns}$

Characteristic Impedance of a coax cable

Assume we are looking at the beginning of a semi-infinite cable.
What is the impedance $\boldsymbol{Z}_0$?



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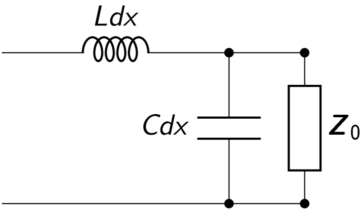

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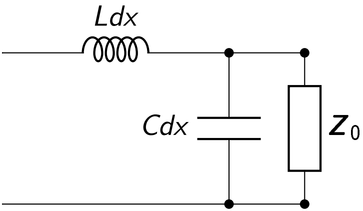

$$\boldsymbol{Z}_0 = j\omega Ldx + \frac{1}{j\omega Cdx + \frac{1}{\boldsymbol{Z}_0}}$$

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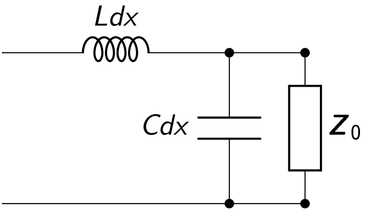

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$$\boldsymbol{Z}_0 = j\omega L dx + \frac{\boldsymbol{Z}_0}{j\omega C dx \boldsymbol{Z}_0 + 1}$$

$$\boldsymbol{Z}_0(j\omega C dx \boldsymbol{Z}_0 + 1) = j\omega L dx(j\omega C dx \boldsymbol{Z}_0 + 1) + \boldsymbol{Z}_0$$

Characteristic Impedance of a coax cable

$$\boldsymbol{Z}_0^2 j\omega C dx + \boldsymbol{Z}_0 = -\omega^2 LCdx^2 \boldsymbol{Z}_0 + j\omega L dx + \boldsymbol{Z}_0$$

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Characteristic Impedance of a coax cable

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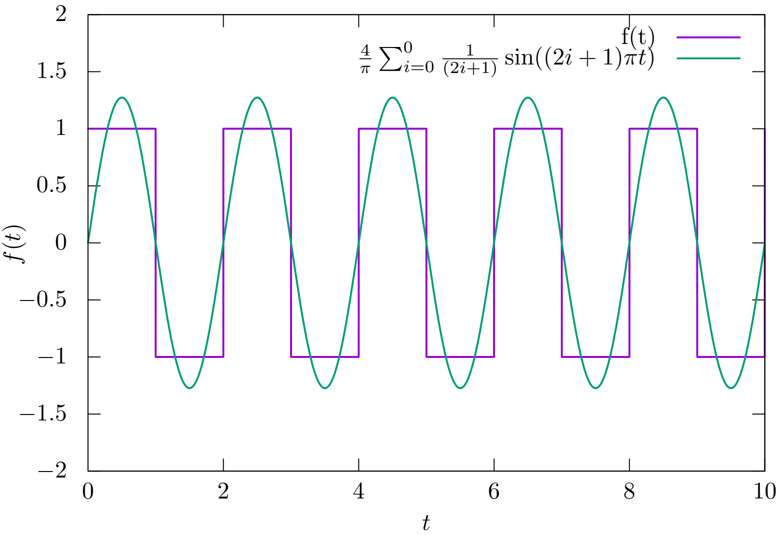
$$\boldsymbol{Z}_0^2 j\omega C dx = j\omega L dx$$

$$\boldsymbol{Z}_0 = \sqrt{\frac{L}{C}}$$

The cable impedance is fully resistive!

Why a sine waves so important?



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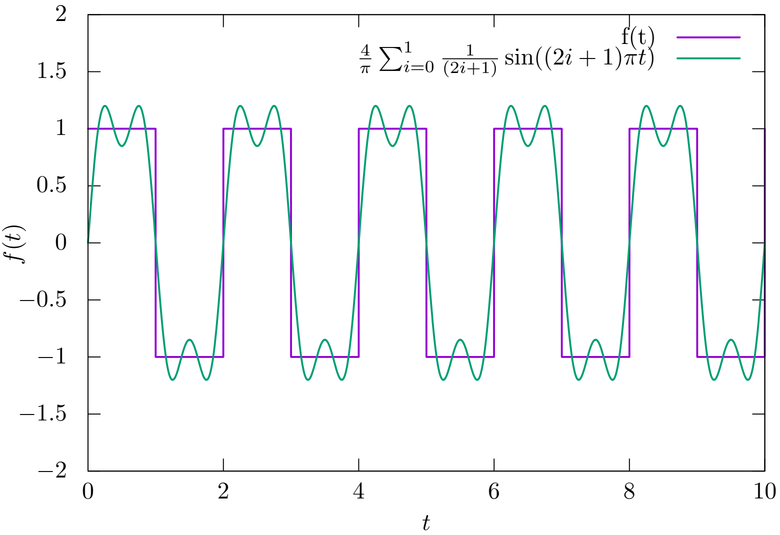

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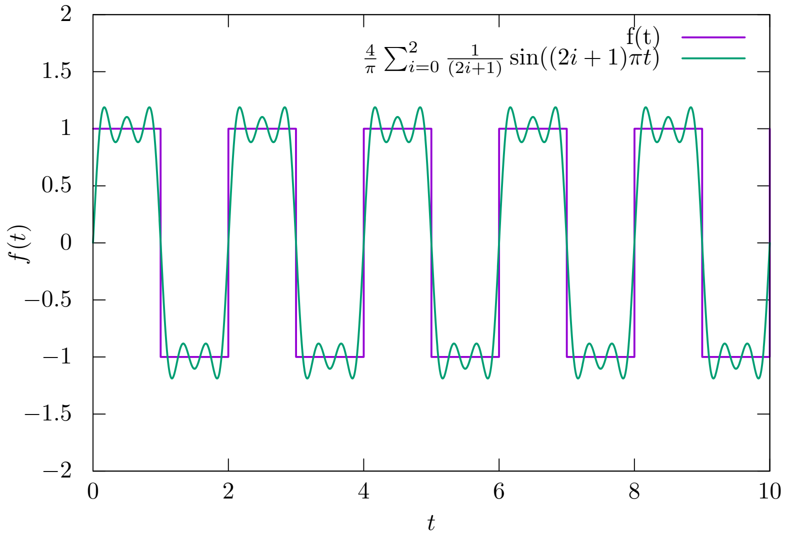
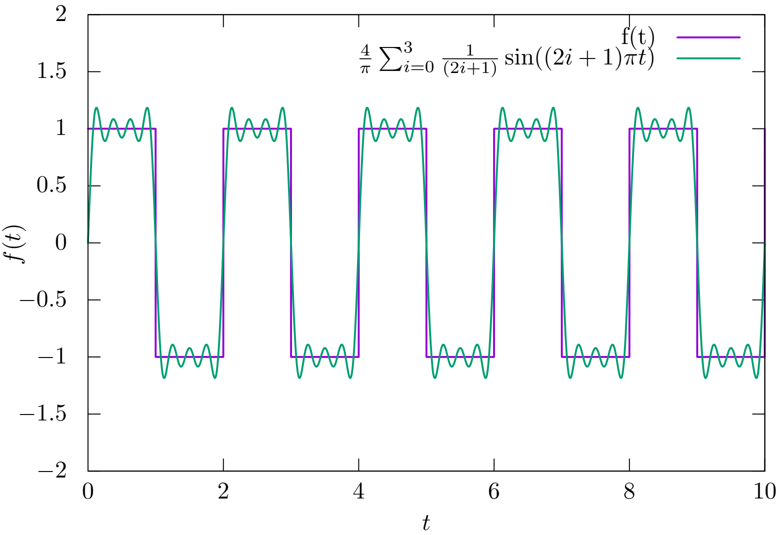


Why a sine waves so important?



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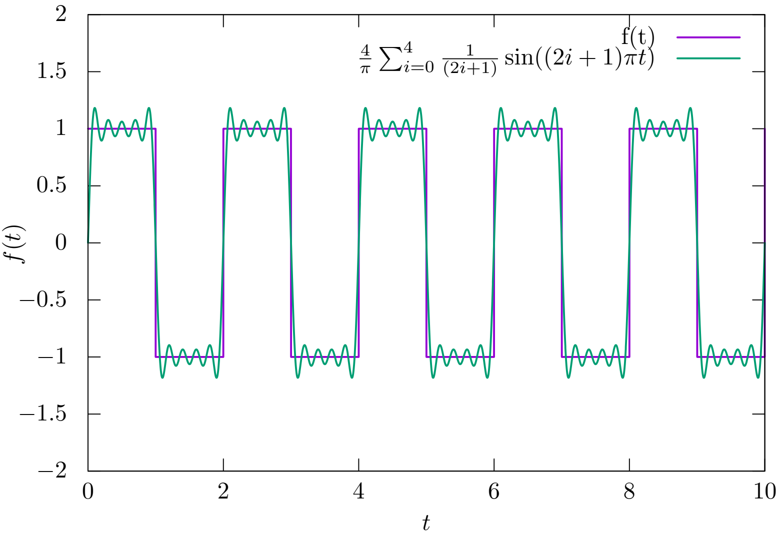

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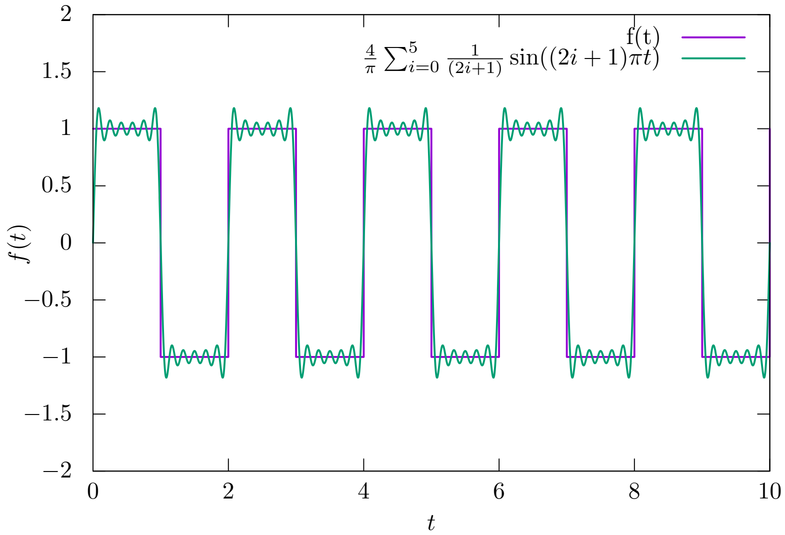

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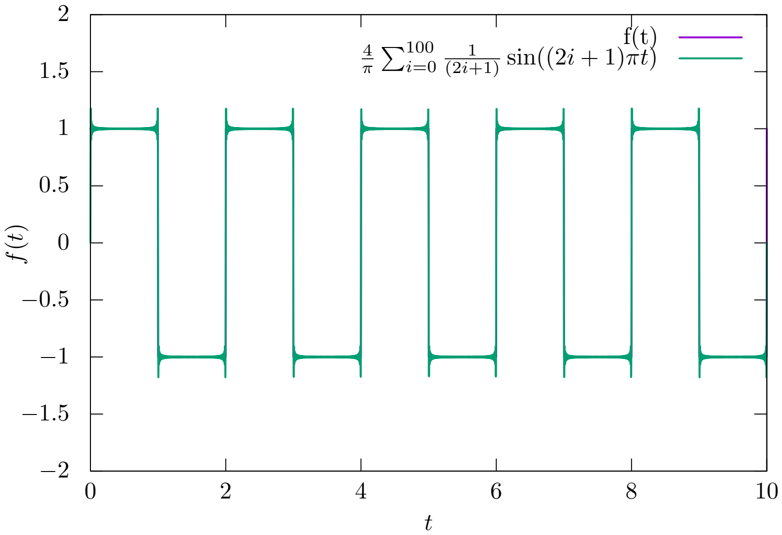


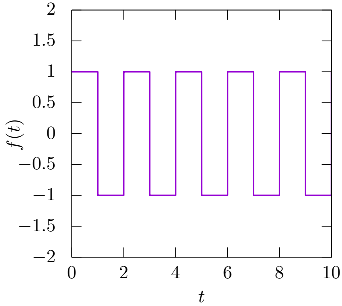
Frequency domain

Every periodic signal f(t) can be associated with a function $F(\omega)$ via the Fourier transform.

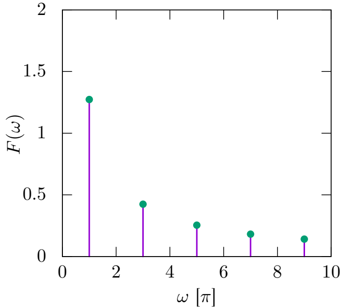
$$\hat{f}(\omega) = \mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \mathcal{F}^{-1}[\hat{f}(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{jt\omega} dt$$

Example



2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
$f(t)$
0
2
4
6
8
10
t



2
1.5
1
0.5
0
$F(\omega)$
0
2
4
6
8
10
ω [π]

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- So either transform V_{in}, build the product, and transform back, or
- Transform $G(\omega)$ and use the Convolution theorem:

$$\mathcal{F}^{-1}[\hat{A}(\omega) \cdot \hat{B}(\omega)] = \int_{-\infty}^{\infty} \bar{A}(\tau)B(t-\tau)d\tau$$