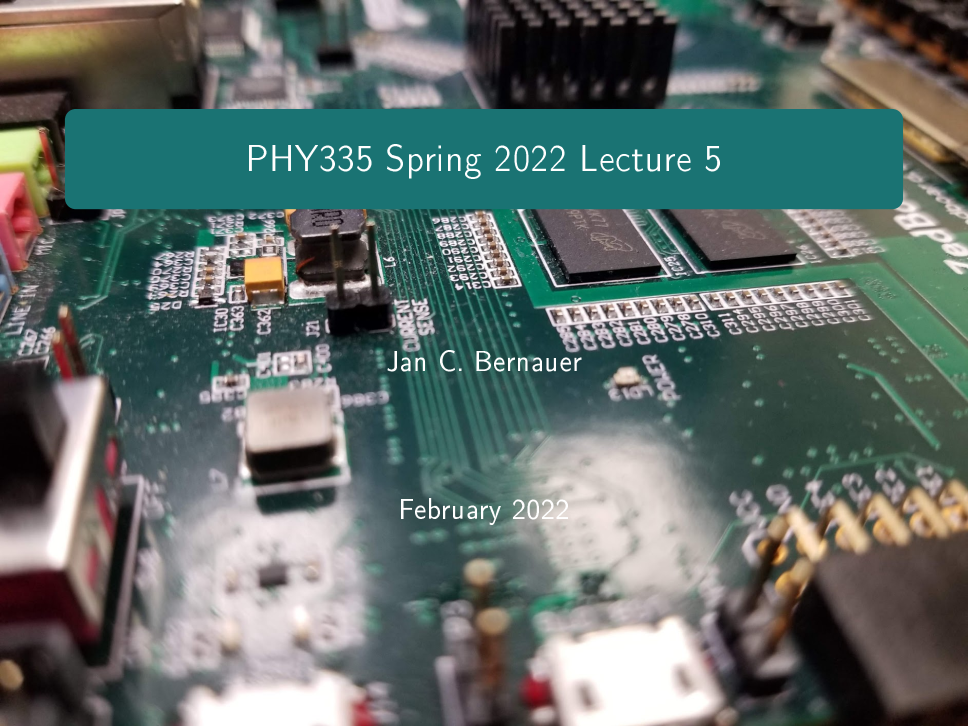

PHY335 Spring 2022 Lecture 5

Jan C. Bernauer

February 2022

Universal amplifier gadget

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What would be the ultimate amplifier gadget?

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Universal amplifier gadget

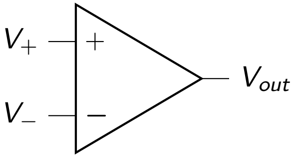
So far, we only could reduce voltages. We need an amplifier!
What would be the ultimate amplifier gadget?

- Arbitrary gain, positive and negative
- Linear
- Infinite input impedance (so we don't load the source)
- Zero output impedance (so we can put arbitrary loads on it)

Ideal op amp

- One output (to the right)
- + is the non-inverting input
- - is the inverting input

Ideal op amp



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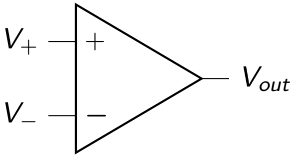

$$V_{out} = A \cdot (V_+ - V_-) = AV_D$$

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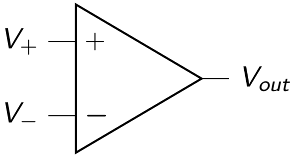


$$V_{out} = A \cdot (V_+ - V_-) = AV_D$$

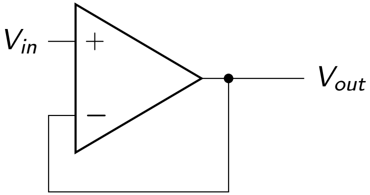
For an ideal op amp, $A = \infty$. So for any any $V_+ \neq V_-$, $|V_{out}| = \infty$

Feedback

Any system where a fraction of the output is fed back into the system is said to have feedback. Feedback is either

- positive: the feedback increases the effective input
  Mostly with catastrophic consequences.
- negative: the feedback reduces the effective input
  This is what we need now!
  (also improves linearity)

Voltage follower



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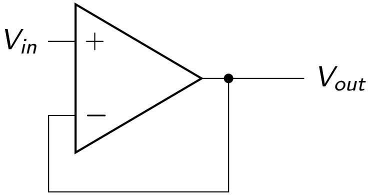

This is obviously negative feedback :)

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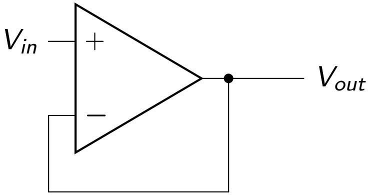

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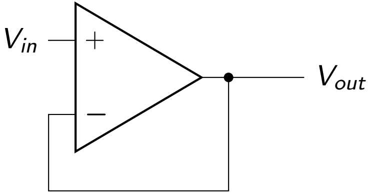
- Let's assume at t=0, $V_{in} = 0$, $V_{out} = 0$

Voltage follower



This is obviously negative feedback :)

- Let's assume at t=0, $V_{in} = 0$, $V_{out} = 0$
- If V_{in} increases, the opamp sees a small voltage difference between it's inputs: $V_D = V_{in} - V_{out}$
- V_{out} will increase until V_D is zero again

Proof

$$V_{out} = A(V_{in} - V_{out})$$

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$$(A+1)V_{out} = AV_{in}$$

$$\frac{A+1}{A}V_{out} = V_{in}$$

For $A \to \infty$

$$V_{out} = V_{in}$$

But why?

No current flowing into the inputs. Output can source arbitrary currents.

That means:

- It appears as infinite resistance: the source of V_{in} is not loaded, i.e. not affected by connecting the voltage follower
- The output voltage is not affected by a load connected to the output

The voltage follower can be used as a buffer, separating a load from an input.

Example for linearity improvement

Let's assume our real opamp is not linear, but still has a large amplification. For example, let's assume

$$V_{out} = 10^5 V_D \times \sqrt{V_D/1V}$$

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$$V_{out} = 10^5 V_D \times \sqrt{V_D/1V}$$

We could now built a $\times 1$ amplifier by setting $V_+ = V_{in}$, $V_- = 0$,and adding a $1 : 10^5$ voltage divider. But then $V_{out} = V_{in} \times \sqrt{V_{in}/1V}$

Example for linearity improvement II

Instead, for a voltage follower with this horrible opamp, we would get:

$$V_{out} = 10^5(V_{in} - V_{out})\sqrt{(V_{in} - V_{out})/1V}$$

Example for linearity improvement II

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$$V_{out} = 10^5 (V_{in} - V_{out}) \sqrt{(V_{in} - V_{out})/1V}$$

$$V_{out}^2 = 10^{10} (V_{in} - V_{out})^2 (V_{in} - V_{out})/1V$$

$$10^{-10} V_{out}^2 1V = (V_{in} - V_{out})^3$$

$$V_{out} + (10^{-10} V_{out}^2 1V)^{1/3} = V_{in}$$

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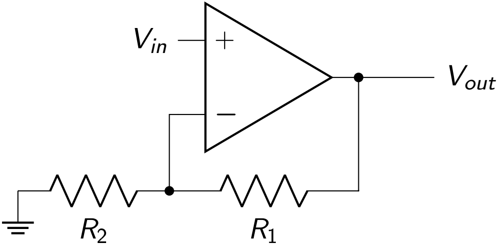
$$V_{out} + (10^{-10} V_{out}^2 1V)^{1/3} = V_{in}$$

Seems pretty linear: 10V output would correspond to 10.002V input instead of $\sim$4.644V

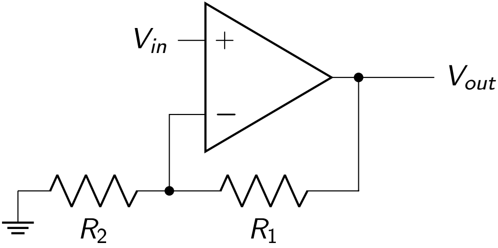
The Golden Rules

- There is no current flowing into the inputs
- In a working circuit with feedback, V_{out} is so that $V_+ = V_-$

Non-inverting amplifier



Non-inverting amplifier



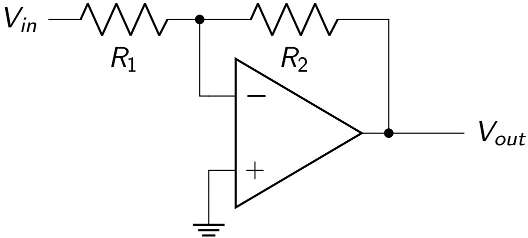
- $V_{in} = V_+$, GR2 $\rightarrow V_{in} = V_-$
- There is no current into the inverting input. Unloaded voltage divider.

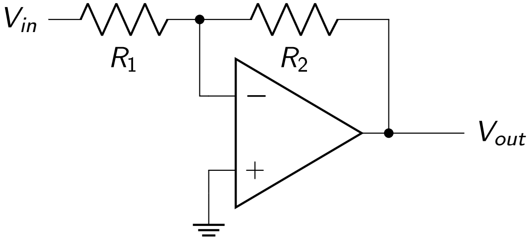
$$V_- = V_{out}\frac{R_2}{R_1 + R_2} = V_{in}$$

- So, voltage gain is

$$G_V = \frac{V_{out}}{V_{in}} = \frac{R_1 + R_2}{R_2}$$

Inverting amplifier





- $V_+ = 0V$, so $V_- = 0V$ (This is called a virtual ground.)
- No current into -, so

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = 0$$

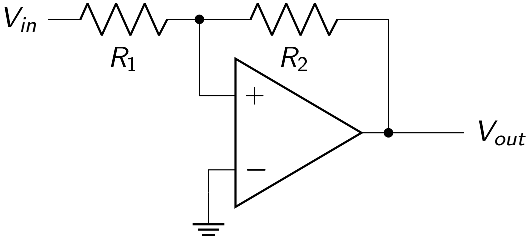
- Voltage gain:

$$G_V = -\frac{R_2}{R_1}$$

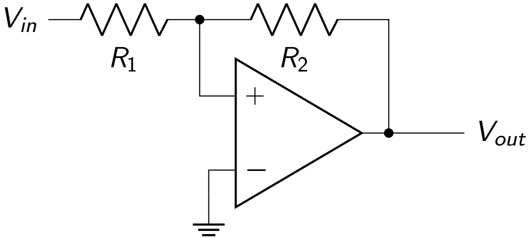
- However:

$$Z_{in} = R_1$$

Non-working inverting amplifier

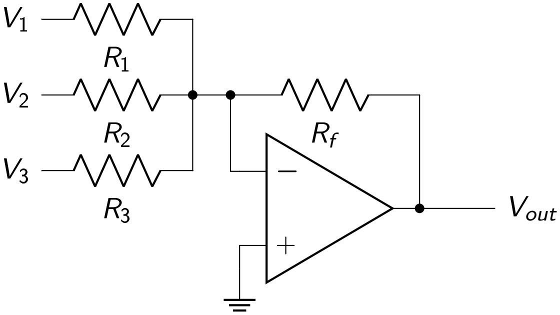


Non-working inverting amplifier

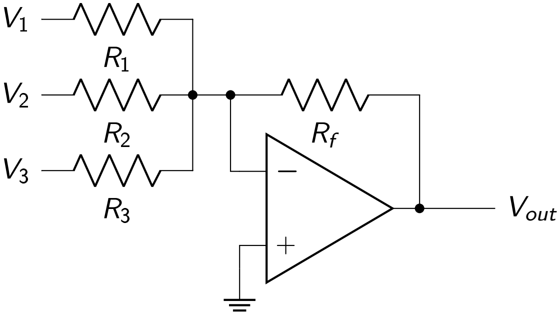


This circuits does not have negative feedback.
Golden rules do not apply!

Voltage adder



Voltage adder



$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_{out}}{R_f} = 0$$

$$V_{out} = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$

If $R_f = R_1 = R_2 = R_3$:

$$V_{out} = -(V_1 + V_2 + V_3)$$

Some notes about real opamps

Real op amps

- have finite amplification
- have amplification which depends on frequency

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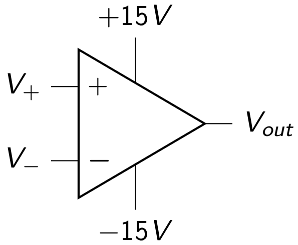
Real op amps

- have finite amplification
- have amplification which depends on frequency
- need power
- can actually not drive that much current
- restrictions on input
- restrictions on output (except rail-to-rail)
- take time to come out of overdrive (output at the min/max)

Power supply

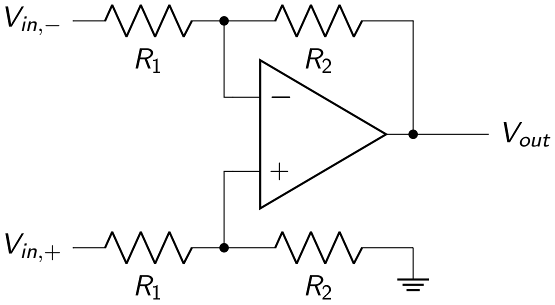
Most often need a split supply: $\pm xV$, often $\pm$15V (exception: single supply op amps). Circuit diagram:



In circuit diagrams, the positive rail is often named V_{CC}, the negative V_{EE}

DANGER: Sometimes, they are also named V_+ and V_-

Differential Amplifier



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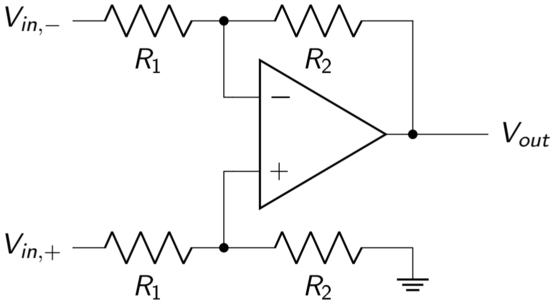
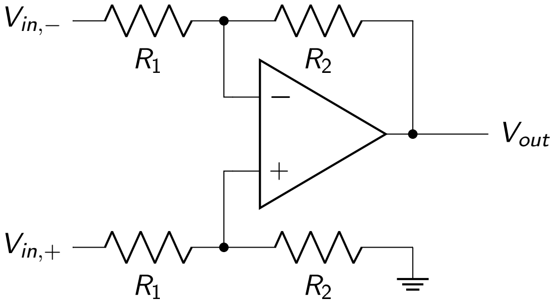


$$V_+ = V_{in,+} \frac{R_2}{R_1 + R_2}$$

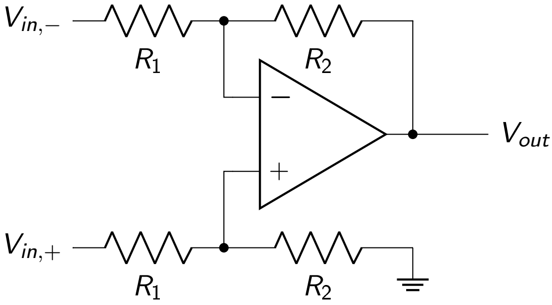
Differential Amplifier



$$V_+ = V_{in,+}\frac{R_2}{R_1 + R_2}$$

$$V_- = (V_{in,-} - V_{out})\frac{R_2}{R_1 + R_2} + V_{out}$$

Differential Amplifier



$$V_+ = V_{in,+} \frac{R_2}{R_1 + R_2}$$

$$V_- = (V_{in,-} - V_{out}) \frac{R_2}{R_1 + R_2} + V_{out}$$

$$V_+ = V_-$$

Differential Amplifier II

$$(V_{in,-} - V_{out})\frac{R_2}{R_1 + R_2} + V_{out} = V_{in,+}\frac{R_2}{R_1 + R_2}$$

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$$(\frac{R_2 + R_1}{R_2} - 1)V_{out} = V_{in,+} - V_{in,-}$$

Differential Amplifier II

$$(V_{in,-} - V_{out})\frac{R_2}{R_1 + R_2} + V_{out} = V_{in,+}\frac{R_2}{R_1 + R_2}$$

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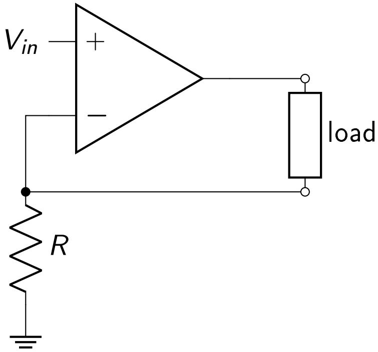
$$(\frac{R_2 + R_1}{R_2} - 1)V_{out} = V_{in,+} - V_{in,-}$$

$$V_{out} = \frac{R_2}{R_1}(V_{in,+} - V_{in,-})$$

Opamp as a (voltage controlled) current source



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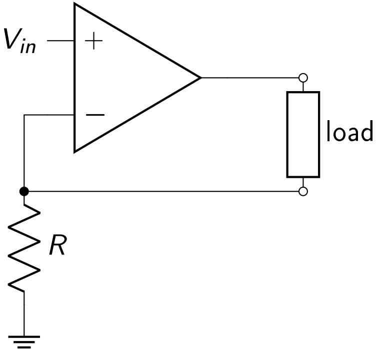
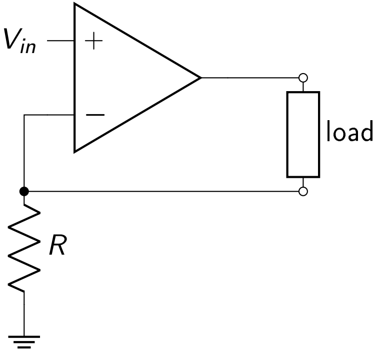


GRs: $V_- = V_+ = V_{in}$

$$I_{load} = \frac{V_-}{R} = \frac{V_{in}}{R}$$

Opamp as a (voltage controlled) current source

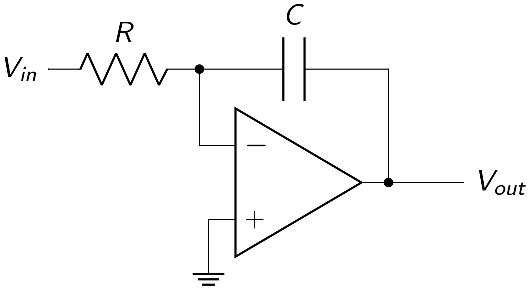


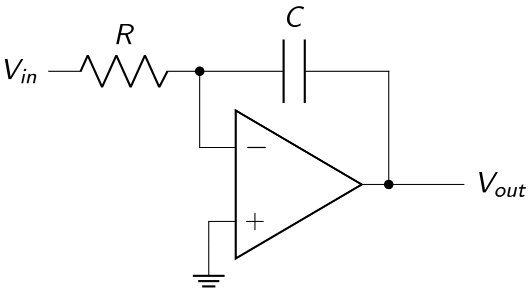
GRs: $V_- = V_+ = V_{in}$

$$I_{load} = \frac{V_-}{R} = \frac{V_{in}}{R}$$

Not ideal: Load does not return to ground.

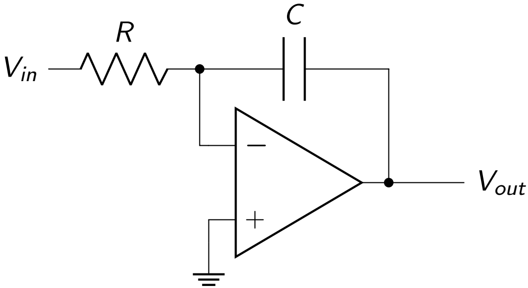
Integrator





$$\frac{V_{in}}{R} = -C\frac{dV_{out}}{dt}$$

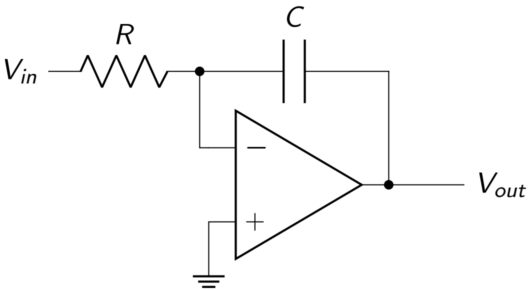
Integrator



$$\frac{V_{in}}{R} = -C\frac{dV_{out}}{dt}$$

$$V_{out}(t) = \frac{1}{RC}\int V_{in}(t) + \text{const.}$$

Integrator



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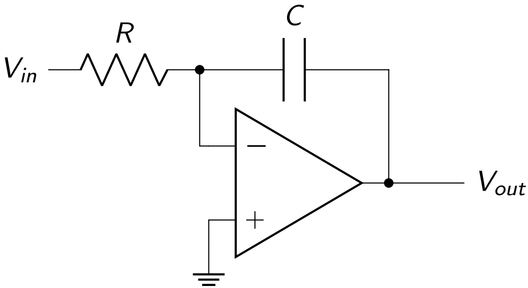
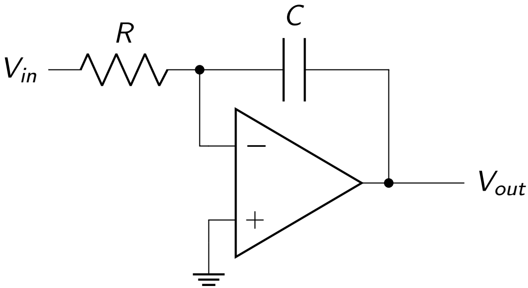
Problem: No feedback for DC


R
C
Vin
Vout
−
+

Integrator



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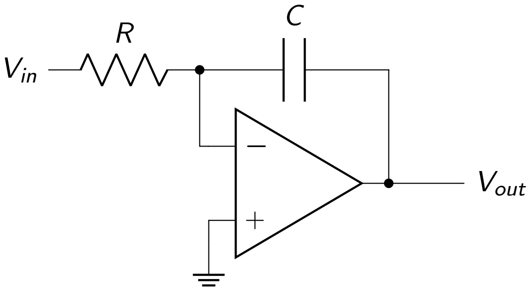

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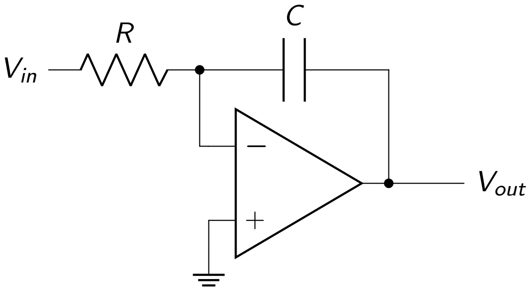


$$\frac{V_{in}}{R} = -C\frac{dV_{out}}{dt}$$

$$V_{out}(t) = -\frac{1}{RC}\int V_{in}(t) + \text{const.}$$

Problem: No feedback for DC. Need to "zero" by shorting out C from time to time.

Integrator, closer look

Let's look at the performance of a real op amp.

- There is some small input current, I_B.
- If V_{in} is not connected, this will produce a voltage drift of $\frac{dV}{dt} = \frac{I_B}{C}$.
- For the TL082, $I_B = 50pA$
- Let's say, for 10nF, we see $\frac{dV_{out}}{dt} = 5mV/s$

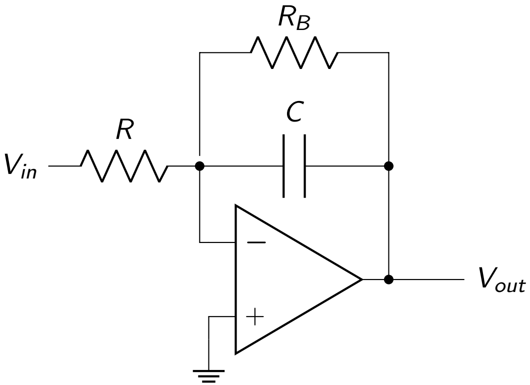
Integrator, closer look II

Let's say V_{in} is actually connected to ground.

- The op amp actually has some input voltage offset, V_{OS} (in the sense that $V_D = V_+ - V_- - V_{OS}$)
- In other words, with $V_- = 0V$, $V_+ = V_{OS}$
- This will produce a current through R
- For the TL082, V_{OS} is 5mV.
- With $R = 1M\Omega$, i.e. 5 nA
- That's a 100 times worse than the error from I_B

Combating drift with a T-network

We can also add a (large) resistor parallel to C to give DC negative feedback.



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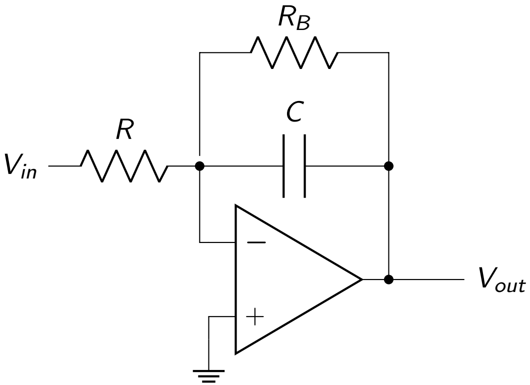
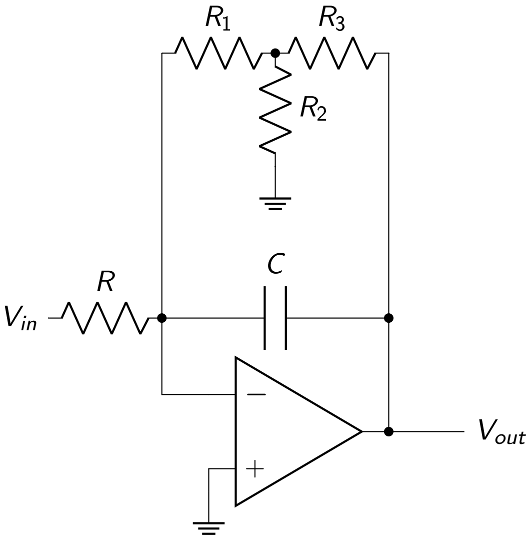
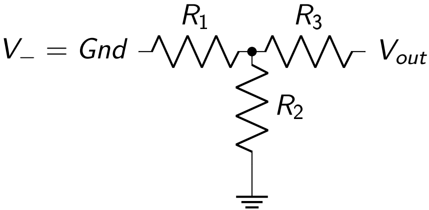


We want to make R_B very, very large. These resistors are hard to come by and have bad parasitic parameters (mainly capacitance)

Combating drift with a T-network II



How does that work?



- R_1 and R_2 both connect the T-node to (virtual) ground.

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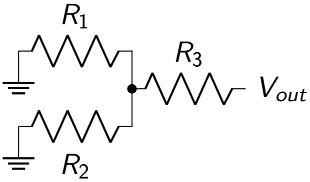

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- $R_2 \ll R_1 = R_3$, which mean $R_3 + R_2 \approx R_3$ and $R1 \parallel R_2 \approx R_2$

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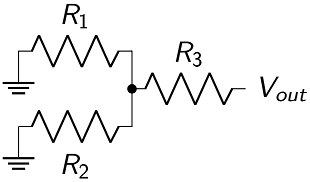
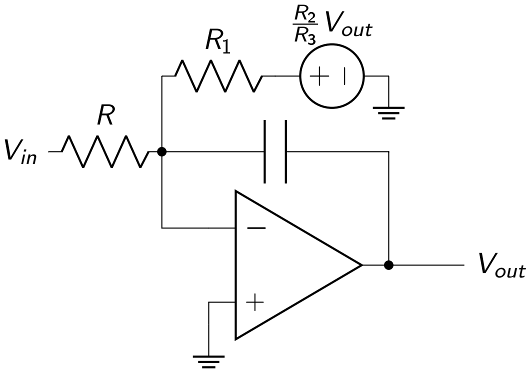


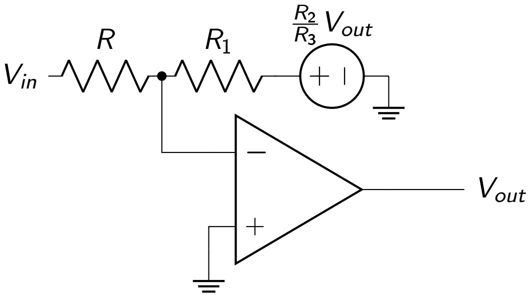
- R_1 and R_2 both connect the T-node to (virtual) ground.
- $R_2 \ll R_1 = R_3$, which mean $R_3 + R_2 \approx R_3$ and $R1 \parallel R_2 \approx R_2$
- So the voltage at the T-node is given by a voltage divider:

$$V_{T-node} = V_{out} \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_3} = V_{out} \frac{R_2}{R_3}$$

On the other hand

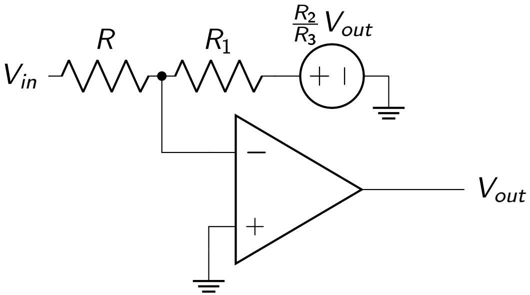


On the other hand



At DC, i.e. $\omega = 0$, we can ignore the capacitor.

On the other hand



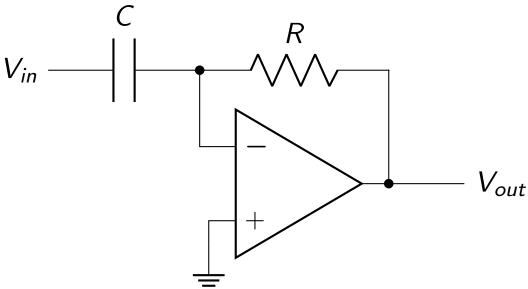
At DC, i.e. $\omega = 0$, we can ignore the capacitor.

$$\frac{V_{in}}{R} = -\frac{V_{T-node}}{R_1} = V_{out}\frac{R_2}{R_1 R_3}$$

The T-Network acts like a large resistor of the value

$$R_T = \frac{R_1 R_3}{R_2}$$

Differentiator



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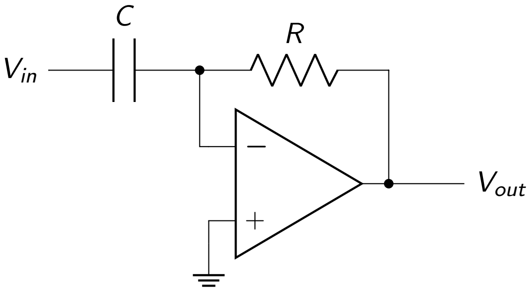
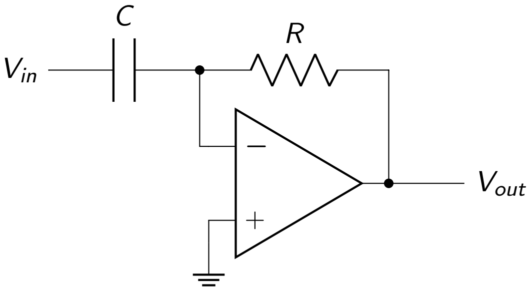


$$C\frac{dV_{in}}{dt} = I = -\frac{V_{out}}{R}$$

Differentiator



$$C\frac{dV_{in}}{dt} = I = -\frac{V_{out}}{R}$$

$$V_{out} = -RC\frac{dV_{in}}{dt}$$

Slew rate

The output of an opamp can only change at a certain, type dependent, maximal rate. This is the so called slew rate SR.

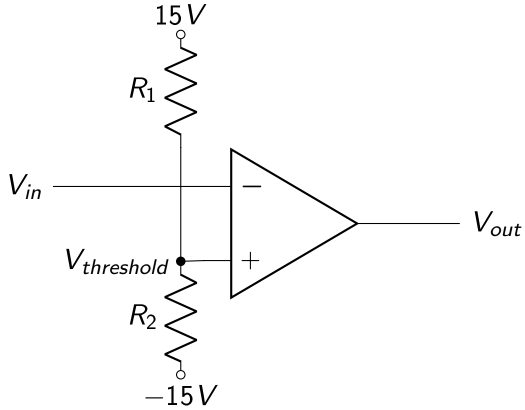
- The slew rate is visible for example if the output should be a square wave, where the voltage level changes are not instantaneous.
- Or as a distortion in a waveform. For a sine wave,

  $$\frac{dV}{dt} = V_0 \omega \cos \omega t$$

  so the frequency at which distortions appear gives the slew rate as $SR = 2\pi V_0 f$

Opamps as comparators

Let's look at a case when no feedback is applied:



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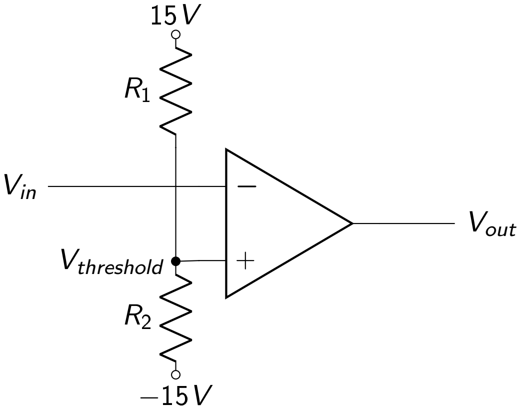
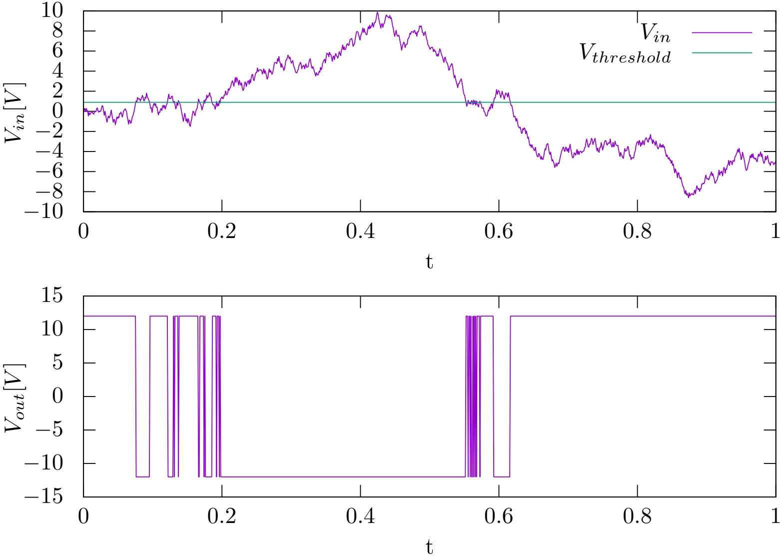
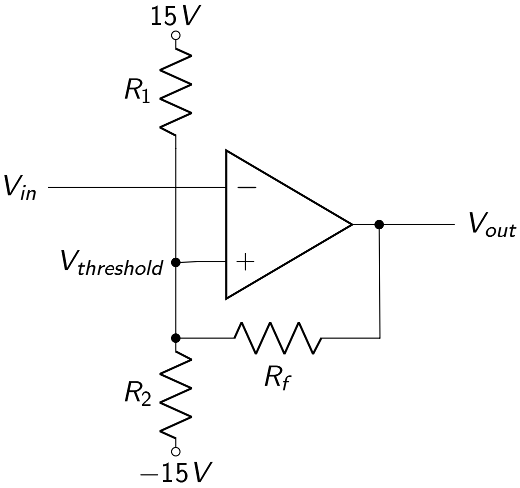


For $V_{in} \neq V_{Threshold}$, the output will saturate at it's minimum or maximum. Optimized opamps for this purpose exist, they are called comparators. Simplest form of an analog to digital converter!

Unstable transition



Adding positive feedback can help:



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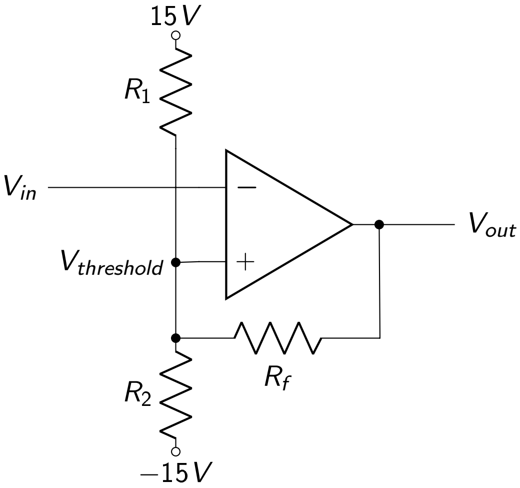
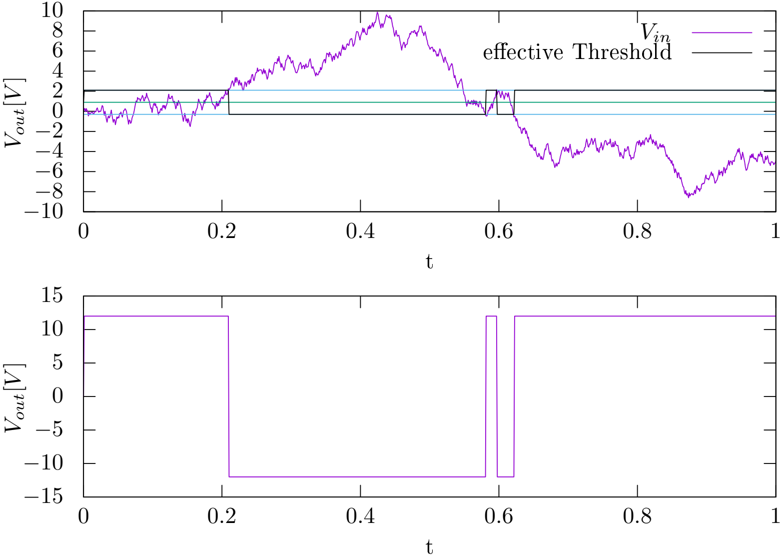


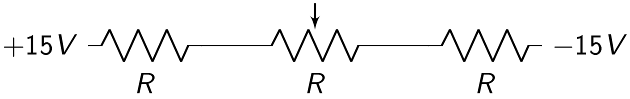
The positive feedback adds hysteresis!

Schmitt-Trigger II



Unit 5 comments

Unit 5, question 2 asks you to build a voltage divider with a potentiometer and two resistors to set a voltage from $\pm 5V$ using a supply of $\pm 15V$.



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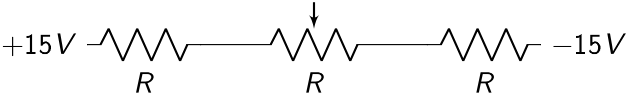
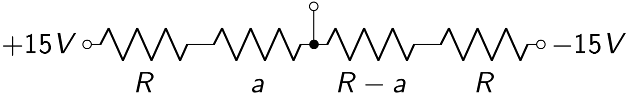
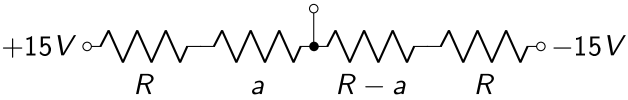


- There are 30V across 3 resistors of the same size. $\rightarrow$ each resistor drops the same voltage, 10V
- One can think of the potentiometer as two resistances which sum up to 10k:



How stiff is this source?



Thevenin equivalent R:

$$R_{Th} = (R + a) \parallel (R + R - a) = \frac{1}{\frac{1}{R+a} + \frac{1}{2R-a}} = \frac{2R^2 + Ra - a^2}{3R}$$

Minimum: $a = 0$ or $a = R$, $R_{Th} = \frac{2}{3}R$.
Maximum: $a = R/2$, $R_{th} = \frac{3}{4}R$

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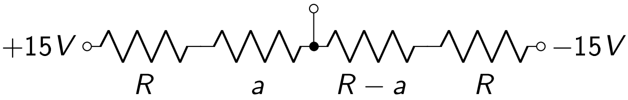

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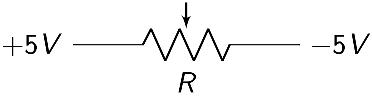
Minimum: $a = 0$ or $a = R$, $R_{Th} = \frac{2}{3}R$.
Maximum: $a = R/2$, $R_{th} = \frac{3}{4}R$
Relative change: $\approx \pm 6\%$

Why not directly to $\pm 5V$?



Now, R_{Th} between 0 and $R/4$. That's $\pm 100\%$