Quantum Computing: Implementing Grover's Algorithm on IBM Q

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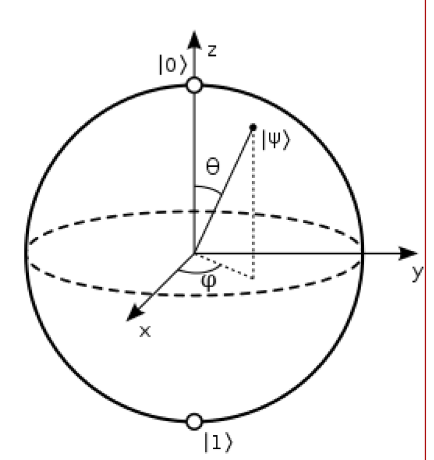
Introduction

- Quantum computers are the future of technology
  - Much faster speeds for certain computations (even exponentially faster)
  - Exploit quantum-mechanical phenomena
- Entanglement: A strong correlation between two individually random particles
- Superposition: Ability to be in multiple distinct states simultaneously
- Unitary Evolution: Basic evolution of the qubit system must be unitary
- Measurement: Collapses the superposition into basis states – for example, a 2-qubit circuit can collapse into $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$.
- Qubits (quantum bits)
  - Can be in superposition of $|0\rangle$ and $|1\rangle$ – Bloch Sphere (below)
  - Subject to noise (environmental disturbances)

Quantum Logic Gates

- X (NOT) gate: $|0\rangle \to |1\rangle$; $|1\rangle \to |0\rangle$
- Hadamard gate: $|0\rangle \to |+\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$;

  $|1\rangle \to |-\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
- Z gate: $|+\rangle \to |-\rangle$; π phase change
- CX (CNOT) gate: If control is $|1\rangle$, X(target)
- CZ gate: If control is $|1\rangle$, Z(target)
- CCX (Toffoli) gate: If both controls are $|1\rangle$, X(target)
- CCZ gate: If both controls are $|1\rangle$, Z(target)



IBM Q

- IBM Q is an initiative to provide public access quantum computers
  - IBM Q 5 Yorktown
  - IBM Q 5 Tenerife
  - IBM Q 16 Rueschlikon
- Experience Documentation – Full User and Beginner Guides
- Composer on IBM Q website for simple quantum circuits

Motivation

- Grover's algorithm is a quantum search algorithm requiring $O(\sqrt{N})$ runtime in contrast to classical $O(N)$ runtime.
- Although only quadratic speedup, Grover's algorithm is vital to countless computer functions which makes it very useful.

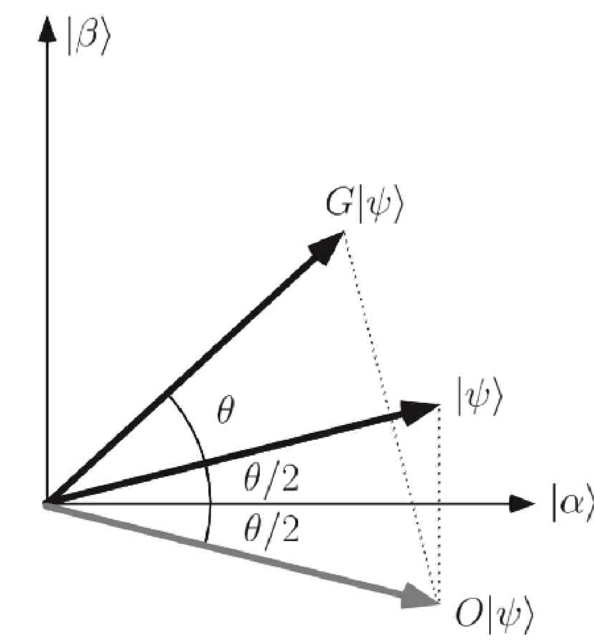
Setup

- Downloaded Anaconda 3 to use Python (3.5)
- Installed QISKit (Quantum Information Science Kit)
- Installed Jupyter
- Downloaded Jupyter tutorials for QISKit
- Learn 2-qubit Implementation of Grover's algorithm and then construct 3-qubit Grover

Grover's Search Algorithm

- **Two steps:**
  - Mark amplitude negative (oracle)
  - Inversion about mean amplitude
- **Assuming $|111\rangle$ is the target**
- **Marking the amplitude of $|111\rangle$ negative**
  - Controlled-controlled-Z

$$\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&-1 \end{bmatrix}$$



- **Inversion about the mean amplitude**
  - $2|\psi\rangle\langle\psi| - I = H^{\otimes n}\,(2|0\rangle\langle 0| - I)H^{\otimes n}$

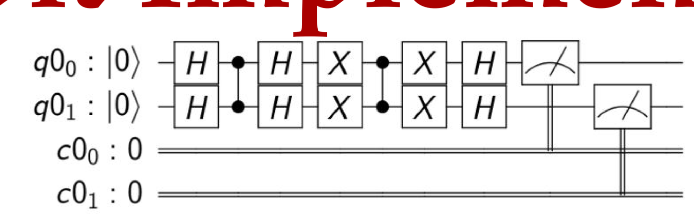
    where $|\psi\rangle = \frac{1}{\sqrt{N}}(|0\rangle + |1\rangle + \cdots + |N\rangle)$
  - $2|0\rangle\langle 0| - I =$

$$2\begin{bmatrix}1\\0\\0\\\dots\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & \dots\end{bmatrix} - \begin{bmatrix}1&0&0&\dots\\0&1&0&\dots\\0&0&1&\dots\\\dots&\dots&\dots&\dots\end{bmatrix} = -\begin{bmatrix}-1&0&0&\dots\\0&1&0&\dots\\0&0&1&\dots\\\dots&\dots&\dots&\dots\end{bmatrix}$$

  - Easily implement $2|0\rangle\langle 0| - I$ using X-gates on each qubit both before and after CCZ gate
- **Number of iterations* = 2**
  - $R \leq \left\lceil \frac{\pi}{4}\sqrt{\frac{N}{M}} \right\rceil = \left\lceil \frac{\pi}{4}\sqrt{\frac{8}{1}} \right\rceil = \lceil 2.22 \rceil = 3$
  - $\theta = \arcsin\left(\frac{2\sqrt{M(N-M)}}{N}\right) = \arcsin\left(\frac{2\sqrt{1(8-1)}}{8}\right) = 41.41°$

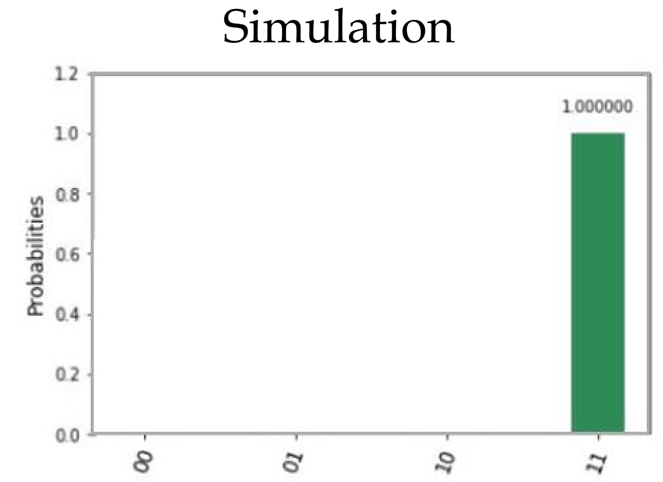
*Nielsen, Michael A., and Isaac L. Chuang. *Quantum Computation and Quantum Information*. 10th Anniversary ed., New York City, Cambridge University Press, 2010.

2-qubit Implementation

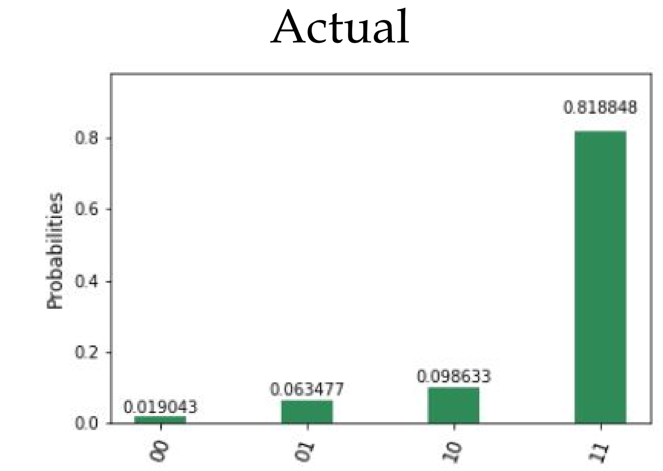


- Assuming $|11\rangle$ is the target
- Oracle uses a CZ gate to mark the amplitude of $|11\rangle$ negative
- Inversion about mean formula
  - H gates around a CZ gate with X gates on each qubit and side
- Only one iteration is required
  - $\theta = \arcsin\left(\frac{2\sqrt{M(N-M)}}{N}\right) = \arcsin\left(\frac{2\sqrt{1(4-1)}}{4}\right) = 60°$
  - A single iteration takes $|\psi\rangle$ directly to $|\beta\rangle$

Simulation



Actual

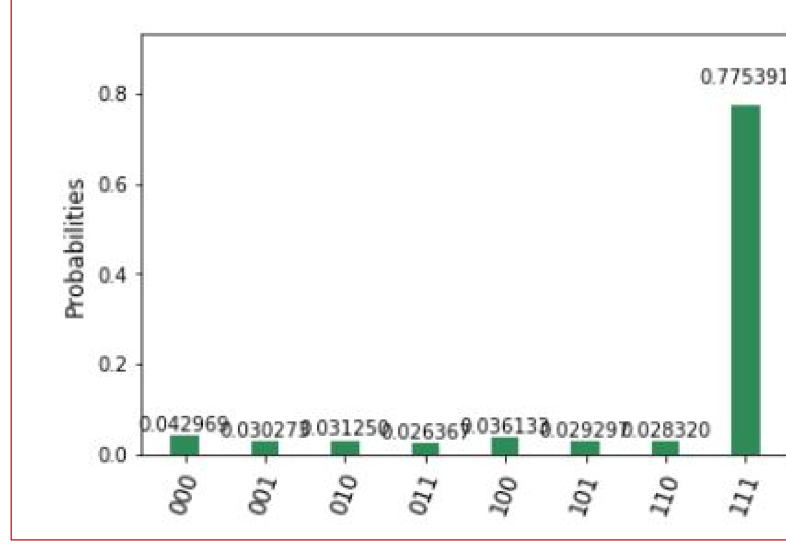


- 2-qubit simulation and actual IBM Q run both returned high probabilities of $|11\rangle$, with some noise affecting the actual run

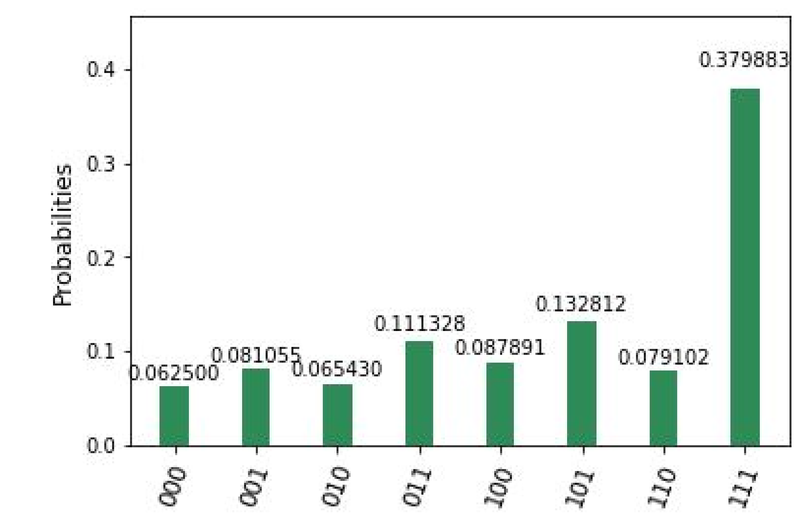
3-qubit Grover: 1 Iteration

- Only one Grover iteration instead of two

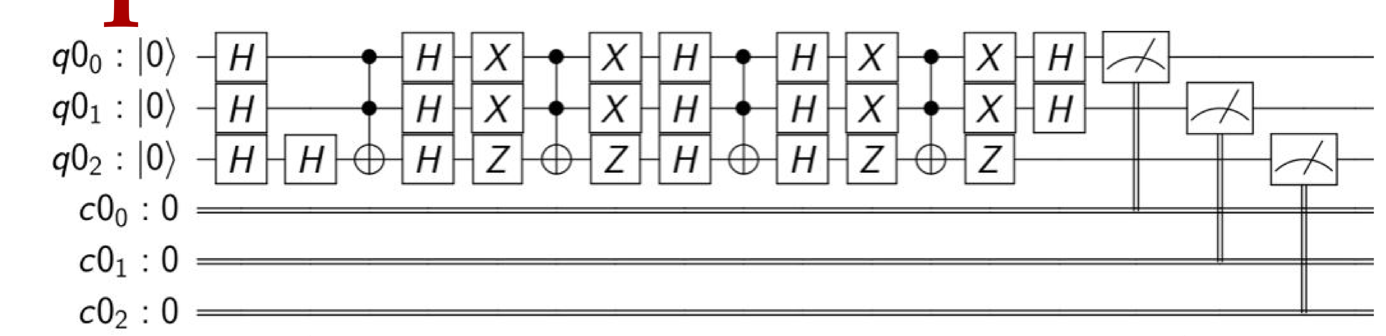
Simulation



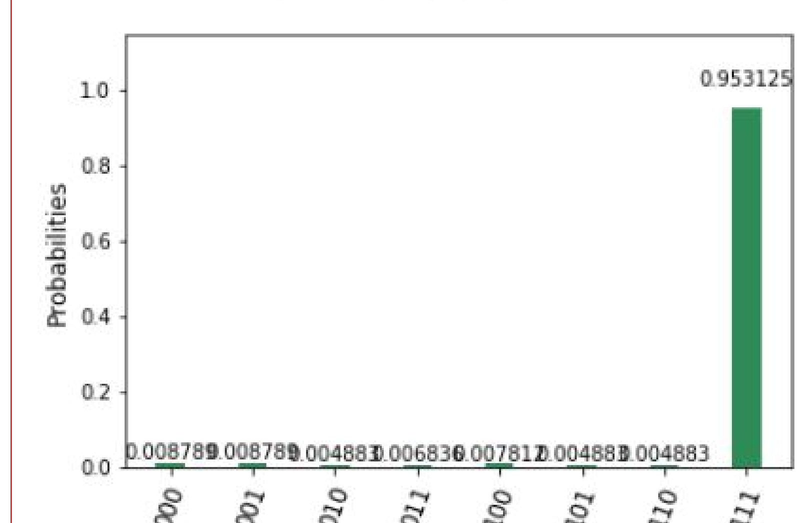
Actual



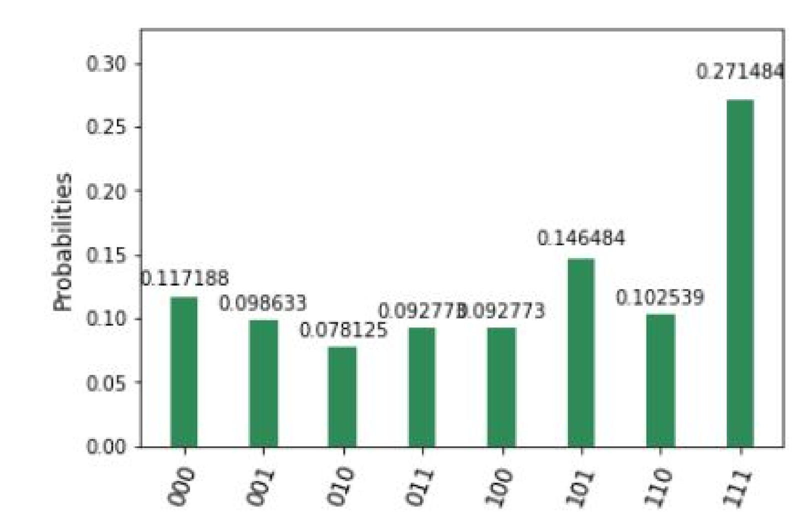
3-qubit Grover: 2 Iterations



Simulation



Actual



Discussion

- Simulation results considerably better than actual results
- 1 Iteration better than 2 Iterations in actual run
- Too much noise in quantum circuit in actual run
  - Toffoli gate needs to be improved; currently implemented using 6 CNOT gates and many single-qubit gates
  - Lose distinction between oracle and inversion about mean
  - May need error correction algorithms
- **Future Work:** Quantum technology research should focus on mitigating effects of noise on circuit

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