Average Connectivity and Average Edge-connectivity in Graphs

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joint work with
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Basic Definitions

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The connectivity and the edge-connectivity of a graph measure the difficulty of breaking the graph apart. However, since these values are based on a worst-case situation, it does not reflect the "global (edge) connectedness" of the graph.

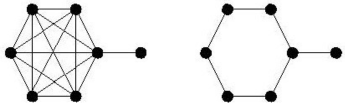

Figure: Two Graphs G_1 and G_2 with $\kappa = \kappa' = 1$

Basic Definitions

The average connectivity of a graph G with n vertices, written $\overline{\kappa}(G)$, is $\frac{\sum_{u,v\in V(G)}\kappa(u,v)}{\binom{n}{2}}$, where $\kappa(u,v)$ is the minimum number of vertices whose deletion makes v unreachable from u.

The average edge-connectivity of a graph G with n vertices, written $\overline{\kappa'}(G)$, is $\frac{\sum_{u,v\in V(G)}\kappa'(u,v)}{\binom{n}{2}}$, where $\kappa'(u,v)$ is the minimum number of edges whose deletion makes v unreachable from u.

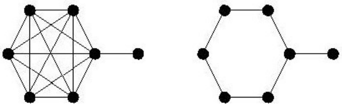

Figure: Two Graphs with $\overline{\kappa}(G_1) = \overline{\kappa'}(G_1) = \frac{27}{7}$ and $\overline{\kappa}(G_2) = \overline{\kappa'}(G_2) = \frac{12}{7}$

Average Connectivity and Matching Number

In 2002, Beineke, Oellermann and Pippert introduced the average connectivity and found several properties of it.

Theorem (Dankelmann and Oellermann 2003)

If G has average degree $\overline{d}$ and n vertices, then $\frac{\overline{d}^2}{n-1} \leq \overline{\kappa}(G) \leq \overline{d}$.

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Theorem (Kim and O 2013)

For a connected graph G, $\overline{\kappa}(G) \leq 2\alpha'(G)$, and this is sharp. Furthermore, if G is connected and bipartite, then $\overline{\kappa}(G) \leq \left(\frac{9}{8} - \frac{3n-4}{8n^2-8n}\right)\alpha'(G)$, and this is sharp.

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- For $vv' \in M$, put v and v' into T, T' and R as follows: If neither v nor v' has a neighbor in S, then put both in T. If v' has a neighbor in S and v does not, then put v in T and v' in T'.

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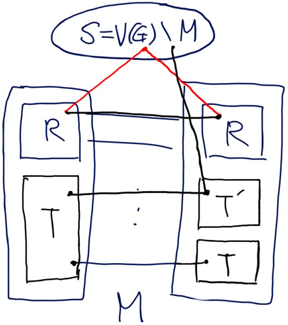

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- Case 1: $u \in S$. If P and P' are distinct internally disjoint u, v-paths, then both of them must visit $V(M) - T$ immediately after u. $\kappa(u, v) \leq 2m - t$.

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- Case 3: $u \in R \cup T$. For the vertex after u on a u, v-path, at most one vertex of S is available. Thus, $\kappa(u, v) \leq 2m$.

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$t = 0$ requires $s = 1$. G is the complete graph with n vertices.

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If G is connected and bipartite, then $\overline{\kappa}(G) \leq \left(\frac{9}{8} - \frac{3n-4}{8n^2-8n}\right)\alpha'(G)$.
This is sharp only for $K_{q,3q-2}$ for a positive integer q.

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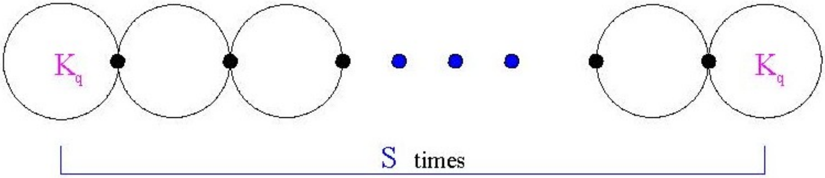
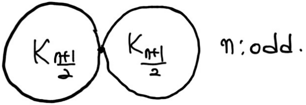


Figure: $\kappa(G) = 1 + O(\frac{q}{s})$ and $\overline{\kappa'}(G) = q - 1$

The above graphs show that there can be a huge gap between average edge-connectivity and average connectivity.

Average Edge-connectivity and Average Connectivity

Question 1. What is the largest gap between the average edge-connectivity and the average connectivity in an n-vertex connected graph?



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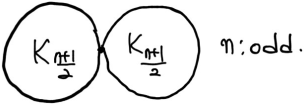
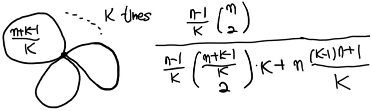


Question 2. What is the largest ratio of the average edge-connectivity and the average connectivity in an n-vertex connected graph?



Average Edge-connectivity in Regular Graphs

An extremal problem: What is the smallest average edge-connecitivity of an n-vertex connected r-regular graph?

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If a graph G has a cut-edge, then we get components after we delete all cut-edges of G. We define an i-balloon to be such a component incident to i cut-edges. Let $B_1 = \overline{P_3 + K_2}$ and let $B_1' = K_4 - e$.

Sketch of Proof

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Sketch of proof: Consider a minimal counterexample G.
$\kappa'(G) = 1$: If not, then $\kappa'(G)\binom{n}{2} \geq 2\binom{n}{2} \geq \binom{n}{2} + \frac{7n+58}{4}$.
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Let G' be the graph obtained from G by replacing D_1 with B_1.

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Sketch of Proof

Theorem (Kim and O 2013)

If G is a connected cubic graph with n vertices, other than K_4, then $\binom{n}{2}\overline{\kappa}'(G) \le \binom{n}{2} + \frac{7n+58}{4}$. Equality holds only for graphs in a special family.

Sketch of proof: Consider a minimal counterexample G.

$\kappa'(G) = 1$:

Every 1-balloon of G is B_1.

Every 2-balloon of G is B_1'.

There are no i-balloons in G for $i \geq 3$.

Questions

Question 3. What is the best upper bound for $\overline{\kappa}'(G)$ in an n-vertex connected r-regular graphs for $r \geq 4$?

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Suppose that r is odd. Let $B_r = \overline{P_3 + \frac{r-1}{2}K_2}$ and $B_r' = K_{r+1} - e$. For odd r, we guess that the graph obtained from the graph in the special family by replacing B_1 and B_1' with B_r and B_r' are the extremal graphs.

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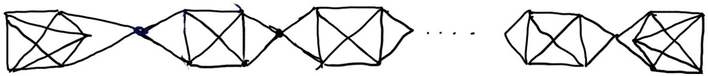

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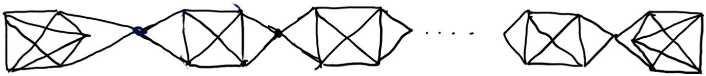

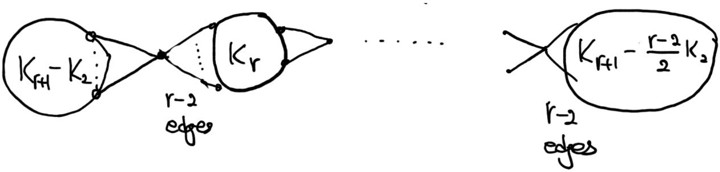

Thank you

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